Visual Perception with Color for Architectural Aesthetics

Michael S. Bittermann
Department of Architecture
Maltepe University
Maltepe-Istanbul, Turkey
michaelsbittermann@maltepe.edu.tr

Özer Ciftcioglu, *Senior Member, IEEE*
Department of Architecture
Delft University of Technology, The Netherlands
Maltepe University, Maltepe - Istanbul, Turkey
o.ciftcioglu@tudelft.nl ozerciftcioglu@maltepe.edu.tr

***Abstract*—Studies on computer-based visual perception and aesthetical judgment for architectural design are presented. In the model, both color and the geometric aspects of human vision are jointly taken into account, quantifying the perception of an individual object, as well as a scene consisting of several objects. This is accomplished by fuzzy neural tree processing. Based on the perception model, aesthetical color compositions are identified for a scene using multi-objective evolutionary algorithm. The methodology is described together with associated computer experiments verifying the theoretical considerations. Emulation of aesthetical judgment is a significant step for applications, where human-like visual perception and cognition are of concern. Examples of such applications are architectural design, product design, and urbanism.**

Keywords*—visual perception; color difference; fuzzy neural tree; architectural design; genetic algorithm; Pareto front

I. Introduction

Visual perception is human's main source of information. Therefore understanding, simulating, or emulating human interaction with environment, inevitably involves this subject. Developing models of visual perception is relevant to the diverse fields where human interaction with environment is concerned, such as cybernetics, robotics, medicine, architecture, and industrial design, making the subject an important one. Perception has been extensively treated in the literature, for instance in philosophy and psychology. Descriptions of the phenomenon in these fields are generally qualitative or statistical in nature. Despite their validity, therefore descriptions of the basic nature of perception are lacking in precision. The same issue also applies to other areas dealing with perception, such as psychophysics [1-3], cognition [4, 5], and image processing [6-11], where the perception concept referred to is generally expressed not mathematically but linguistically. For instance, in the psychophysics and cognition works, brain processing in human visual system is explained via neurobiological terms rather than mathematical ones. Yet, establishing a model of human perception implies that the phenomenon should be treated in computational form for minimal ambiguity. Although image processing studies are a matter of computation, and they traditionally do make reference to biological vision in order to justify or have inspiration in the development of machine vision algorithms, the algorithms resemble to human vision only in a restricted sense. Generally an image processing algorithm singles-out a component of perception process occurring in human visual system. Examples are the ample edge detection studies in the literature, e.g. [12, 13], and works on recovering three-dimensional object information from two-dimensional image data, e.g. [14, 15]. However, due to the specific nature of the image processing applications, there is no need that the computations reflect some general characteristics of human vision that are due to the totality of interrelated brain processes. One of the most observable of such characteristics is the uncertainty of remembering visual information.

When an observer pays visual attention to multiple objects existing in his visual scope, there is likelihood the observer does not notice the presence of some objects, i.e. he is unable remember them. The cause of the phenomenon is the complexity of human visual system. Dealing with applications that need not closely resemble human activities, image processing works generally can afford to ignore such complexity induced properties. Hence, the works generally do not refer to human vision as the object of the modeling effort. The complexity is due to the multiple, interrelated, and merely partially understood brain processes that are involved in perception. Therefore probabilistic treatment of the phenomenon is most convenient, as the imprecisions in description of the concept are subject to absorption in a probabilistic model. In order to delineate the probabilistic treatment in this work from existing probabilistic treatment in vision related research, one notes that in object detection studies perception is considered to be the engagement of pattern recognition. In this case Bayesian methods are appropriate [16]. However, for human perception modeling Bayesian approach turns out to be trivial [17]. This is because for human the probability of a retinal image given a certain scene is almost certain, which implies that the recognition of the scene given a retinal image is also almost certain. Therefore, as the subject matter of this work is human perception, notwithstanding the validity of the Bayesian approaches for computer vision they are of minor importance here.

In this work two causes of the uncertainty in visual perception are considered. The first cause is that in a scene with multiple objects, visual attention is only partly devoted to each object, so that the amount of attention paid for an object has a likelihood of being insufficient for yielding the remembrance of the object. An object subtending a larger portion of our visual field implies a greater likelihood to be seen, and hence remembered. This phenomenon has been treated in the literature [18]. The second cause is that an object having very similar color with objects behind and in front of it may not standout enough from its surrounding to be noticed. Clearly, a greater color difference between an object and its background implies a greater likelihood we see and remember the object. Modeling of visual perception including both, the geometric and the color aspects, is missing in the literature. This work tackles this issue by fusing geometric and color perceptual information. The fusion is possible, since both perceptual properties are treated as likelihoods in this work, while they have alternative interpretations as fuzzy memberships. Therefore, the fusion is executed by means of a likelihood-based fuzzy computation, quantifying the intensity of a perception in the form of likelihood. This is the first item addressed in this study. Based on the first one, the second item is pinpointing the role of the intensity of perception in aesthetical judgment of the color composition of a scene. Based on these considerations, the reason why certain color compositions of a scene strike an unbiased observer as aesthetical is investigated. The organization of the paper is as follows. In section II, computation of the likelihood of visual perception is presented. Based on the perception computations, the multi-dimensional nature of color aesthetics is exposed in section III. The validity of the model is verified by means of computer experiments in section IV. This is followed by conclusions.

II. Likelihood of Visual Perception

One source of the uncertainty characterizing visual perception is due to the geometry of an object subject to perception in relation to visual scope of observation. This has been treated in the literature [18], where the visual attention is modeled as a uniform probability density (pdf) with respect to solid vision angle Ω_S defining the visual scope. In that work the visual attention paid to environment is modeled by

$$f_\Omega(\Omega) = 1/\Omega_S \qquad (1)$$

Based on this, the perception of an object that is due to its geometric presence is defined as the integral of the attention over the solid angle domain subtended by the object and given by the probability

$$p_{obj} = \int_0^{\Omega_{obj}} \frac{1}{\Omega_S} d\Omega = \Omega_{obj} / \Omega_S \qquad (2)$$

Next to geometry, the difference in color between an object and the objects surrounding it is a second important condition to be fulfilled for perceiving an object. For computing color difference, first it is imperative to represent a color numerically. This is the subject matter of colorimetry. In extensive color matching experiments, the Commission Internationale de l'Eclairage (C.I.E.) established the means to represent by vectors of three numbers the set of colors a standard human observer is able to perceive [19, 20]. The experiments rely on the law of *trichromatic generalization*, which states that (i) stimuli with same specifications look alike to an observer with normal colour vision under the same observing conditions, (ii) stimuli that look alike have the same specification, and (iii) the numbers comprising the specification belong to continuous functions [21]. In such matching experiments a large group of observers is asked to produce a certain colour by adjusting separately the intensity of three monochromatic primary colours red, green and blue with a known wavelength. The observers should combine the three colours in such a way that the combination matches as close as possible to a given monochromatic test sample. The colours are typically presented in the two halves of a bipartite visual field. This way any influence of geometry in the colour matching is minimized. The three numbers resulting from the conversion of a light stimulus using C.I.E.'s standard observer model [22] are termed as *tristimulus values.* In their normalized from they are referred to as *chromaticity coordinates* [23].

Estimating the difference between two colors in the context of perception modeling requires that the difference quantity obtained matches to the difference a standard human observer would attribute to the color pair. This is conveniently accomplished when the color space, in which the pair is represented, is perceptually uniform. This property stipulates that the Euclidian distance between the chromaticity coordinates of two colors quantifies the color difference attributed by human for these colors. The first color spaces introduced by C.I.E., namely the *1931 C.I.E. RGB* space and a transformed version of it named *1931 C.I.E. XYZ* space, have both been shown to lack in perceptual uniformity [24]. To alleviate this drawback, C.I.E. introduced two approximately uniform spaces by two different transformations of the *XYZ* space, known respectively as *CIE 1976* $L^*u^*v^*$ and *CIE 1976* $L^*a^*b^*$ spaces [22, 25]. The L^* component in either space corresponds to the lightness of a color. $*u*v*$ and $a*b*$ are chromaticity coordinates, which, in combination specify the saturation and hue of a color. Detailed definitions of these quantities can be found here [21]. It is emphasized that due to the approximate perceptual uniformity of both spaces, color differences in either space are obtained by the Euclidian distance among the chromaticity coordinates. Explicitly, the color difference ΔE between two colors $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ in $L^*a^*b^*$ space is obtained by

$$\Delta E^*_{ab}(\boldsymbol{c_1}, \boldsymbol{c_2}) = \sqrt{(L^*_2 - L^*_1)^2 + (a^*_2 - a^*_1)^2 + (b^*_2 - b^*_1)^2} \qquad (3)$$

One notes that the uniformity in the distance computation for small color differences, i.e. $\Delta E<10$, has been further improved by the C.I.E. due to [26]. The difference ΔE_1 between two colors $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ having chromaticity $a\neq 0$ and $b\neq 0$ is generally larger compared to the distance ΔE_2 between their achromatic, i.e. grey, counterparts denoted by $\boldsymbol{c'_1}$ and $\boldsymbol{c'_2}$. This is due to the fact that color difference between achromatic colors, is bound to be exclusively along the lightness dimension. Converting a chromatic color to its achromatic counterpart means that the chromaticity coordinates a^* and b^* are set to zero, so that the color is without any chroma. That is, a chromatic color is projected parallel to the a^*b^* plane onto the L^* axis, where its achromatic counterpart lies. Referring to (1), with the exception of the trivial case $\boldsymbol{c_1}=\boldsymbol{c_2}$, since

$$\sqrt{(L^*_2 - L^*_1)^2 + (a^*_2 - a^*_1)^2 + (b^*_2 - b^*_1)^2} < L^*_2 - L^*_1, \quad \boldsymbol{c_1} \neq \boldsymbol{c_2} \qquad (4)$$

therefore clearly

$$\Delta E^*_{ab}(\boldsymbol{c_1}, \boldsymbol{c_2}) < \Delta E^*_{ab}(\boldsymbol{c'_1}, \boldsymbol{c'_2}), \quad \boldsymbol{c_1} \neq \boldsymbol{c_2} \qquad (5)$$

This explains why chromatic images are generally more memorable compared to their achromatic counterparts.

Investigating the role of color difference in perception, as a simple case we consider the situation of a single occlusion, namely a single object in front of a background. The background of course also has a color, which can be achromatic one in some cases. The likelihood the object fulfills the conditions for perception due to color is characterized by the normalized color difference denoted by $\Delta E^*_{ab\_n}$ and given by

$$L_{c\_obj} = \Delta E^*_{ab\_n}(\boldsymbol{c_1},\boldsymbol{c_2}) = \frac{\Delta E^*_{ab}(\boldsymbol{c_1},\boldsymbol{c_2})}{\Delta E^*_{ab\_\max}} \tag{6}$$

where $\Delta E^*_{ab}(\boldsymbol{c_1}, \boldsymbol{c_2})$ denotes the color difference between the object's color $\boldsymbol{c_1}$ and the color of the background $\boldsymbol{c_2}$, *and* $\Delta E^*_{ab\_max}$ denotes the maximal color difference between two colors, both expressed in $L^*a^*b^*$ color space coordinates. $\Delta E^*_{ab\_n}$ is the likelihood that the color specific condition for perceiving an object is fulfilled, namely some physical variation manifesting the existence of an object with distinct material makeup exists. In case an object has exactly the same color as its background, then the physical variation is close to zero and the likelihood that the color condition for perception of the object is fulfilled, vanishes. Conversely, if an object has the maximally possible color difference in perceptually uniform color space, then the likelihood, that the condition for perceiving the object due to its color is fulfilled, is maximal namely unity. It is emphasized that, as any color difference represents a possible fraction of $\Delta E^*_{ab\_max}$, therefore (6) expresses the likelihood that the condition to perceive the object due to color is fulfilled. It is noted that in (6) $\Delta E^*_{ab\_max}=375.6$ when the entire visible spectrum of colors is considered. Color reproducing devices, such as computer monitors, however are not capable of displaying the entire visible spectrum, but merely a restricted portion of it. The portion is referred to as the device's gamut. Therefore, due to the color limitations of the monitor, in computer-based applications ΔE_{ab_max}<<375.6, in general. Let us consider another situation, where there is a second object located behind our first one, and in front of the background. In case the second object is larger than object nr. 1 and located in such a way that nr. 1 has no color difference vis-à-vis the background anymore, but only vis-à-vis the second object, then ΔE_{ab_n} of object nr. 1 is given as before by (3), where this time $\boldsymbol{c_1}$ denotes the color of the first object and $\boldsymbol{c_2}$ the color of the second object.

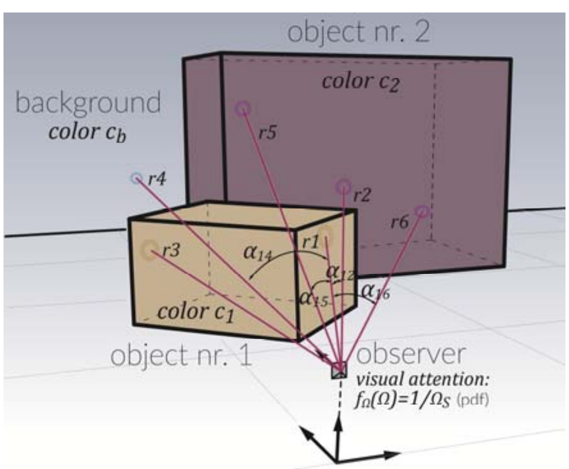


Fig. 1. Rays simulating visual perception of the color differences along the perceived limitation of object nr. 1 vis-à-vis object nr. 2, as well as object nr. 1 vis-à-vis the background.

The situation becomes more involved and interesting when the second object is located in such a way that part of the first object's perception is still vis-à-vis the background, as seen in figure 1. Such partial occlusion is the general case in everyday perception, and one notes that the two situations described just before are special cases of this general one. More explicitly, in general an object may be partly occluded by another object, or may itself partly occlude another object. In this case multiple color differences need consideration in order to compute the likelihood that the object fulfills the color related perception condition due to its visual context. Every color difference vis-à-vis every partly occluding and occluded object needs to be taken into account in this abstraction. The geometric perceptions of the occlusion regions play the role of weights in the computation of the mean color difference $\Delta\bar{E}^*_{ab}$ given by

$$\begin{aligned} L_{c\_obj} &= \Delta\overline{E}^*_{ab\_n\_obj} = \sum_{i=1}^{i=nr} w_i \Delta E^*_{ab\_n}(\boldsymbol{c}_{obj},\boldsymbol{c}_{occ\_i}) \\ &= \sum_{i=1}^{i=nr} [\frac{p_{r\_occ\_i}}{\sum_{j=1}^{j=nr} p_{r\_occ\_j}} \Delta E^*_{ab\_n}(\boldsymbol{c}_{obj},\boldsymbol{c}_{occ\_i})] \end{aligned} \tag{7}$$

where the weight factors are the normalized perceptions belonging to each region where an occlusion involving the object *obj* occurs. The perceptions are denoted by p_r; *nr* denotes the number of regions where occlusions vis-à-vis *obj* occur. Comparing (6) and (7) one notes that (6) is a special case of (7) for $nr=1$. The justification of (7) is clear from consideration of the extremity of the expression. When a certain partial occlusion is hardly noticeable due to the geometric constellation, i.e. $p_{r\_obj,obj\_i}\approx 0$ it is clear that then the color difference belonging to this particular occlusion has hardly any influence on the fulfillment of the color difference condition and hence on perception as well. To illustrate (7) let us consider object nr. 1, as it is perceived from the observer in the figure. The significance of the color difference the object has with the background is comparable with that between object nr. 1 and object nr. 2. This is because the two occlusion regions, the one between object nr. 1 and the background and the one between object nr. 1 and object nr. 2, each have approximately the same quantity of geometry induced perception given by (2). Hence, $L_{c\_obj1}\approx 0.5\Delta E^*_{ab\_n}(\boldsymbol{c_1},\boldsymbol{c_b})+0.5\Delta E^*_{ab\_n}(\boldsymbol{c_1},\boldsymbol{c_2})$. The situation might be quite different for object nr. 2. The two occlusion regions, the one between object nr. 2 and the background, and the one between object nr. 2 and object nr. 1, apparently have quite different quantities of perception.

Due to the complexity of geometric constellations of objects in visual scope, analytical treatment of the situation is inconvenient. In particular computing the normalized p_r in (7) belonging to different partial occlusion regions using (2) is a problematic issue, since the solid angle of the occlusion region is ill defined. To handle the situation we use a probabilistic ray tracing approach as follows. To simulate the color difference perception we are sending rays away from the point of observation in random directions within visual scope. One notes that the probability density function (pdf) modeling visual attention is constant per differential solid vision angle Ω as given by (1). In figure 1, six rays are shown denoted by r_1, .., r_6. Each vision ray intersects either object nr.1, object nr. 2, or the background as the nearest object along the ray. Considering object nr. 1 the color difference along the object's occlusion region is obtained by the following procedure. For every ray that intersects object nr. 1, for instance r_1, we compute the angles between this ray and all other rays that did not intersect object 1. In the figure this is exemplified for ray r_1 and the respective angles are denoted by α_{12}, α_{14}, α_{15}, α_{16}. The smallest one among the angles is identified, which is α_{12} in the figure, and its associated ray r_2 is considered as member of the set of rays simulating the color difference perceived at the occlusion region of object nr. 1. We term such rays as *occlusion rays* of object 1. As r_2 is intersecting object nr. 2, therefore the color difference associated with the ray pair r_1r_2 is the most likely representative of the difference between the colors of object nr. 2 denoted by c_2 and object nr. 1 denoted by c_1. The color differences are obtained between every ray intersecting object nr. 1 and its respective associated occlusion ray, and normalized as given by (6). The individual color differences obtained for all occlusion rays are summed up and divided by the total number of occlusion rays. This way a the mean color difference perceived for the object $\Delta\bar{E}^*_{ab}$ is obtained by

$$L_{c\_obj} = \Delta\overline{E}^*_{ab\_obj} = \frac{1}{n}\sum_{r=0}^{r=n}\Delta E^*_{ab\_n}(\boldsymbol{c}_r, \boldsymbol{c}_\rho) \qquad (8)$$

where n is the number of occlusion rays surrounding the object, c_r denotes the color of the object that is intersected by ray *r, and* c_ρ is the color of the object that is intersected by the ray subtending minimum angle with r. In the example in the figure the only other ray intersecting object nr.1 is r_3, and its corresponding occlusion ray is r_4. For the example $\Delta\bar{E}^*_{ab\_Obj}$ in (8) is obtained by $[\Delta E_{ab\_n}(\boldsymbol{c_1}, \boldsymbol{c_2}) + \Delta E_{ab\_n}(\boldsymbol{c_1}, \boldsymbol{c_b})]/2$. The accuracy of $\Delta\bar{E}^*_{ab\_Obj}$ is dependent on the number of rays that are used to simulate the vision, and it can be raised to an arbitrary value, limited exclusively by the available computation time.

Fusion of the geometric and color perception information is accomplished in this work using the likelihood-based neural processing method, known as fuzzy neural tree (FNT), described in [27]. The rationale to use the approach is that the imprecision of the perceptual information is to treat by means of soft computing methodology. The categorization into geometric and color aspects in perception is an act of human linguistic abstraction that it is best dealt with the methods of soft computing. In particular, the imprecision in perception has a probabilistic-possibilistic nature that is uniquely dealt with by the FNT method. In a fuzzy neural tree, the output of *i*-th terminal node is denoted x_i and it is introduced to a non-terminal node. The detailed view of node connections from terminal node *i* to internal node *j* and from an internal node *i* to another internal node *j* is shown in the publication [27]. The connection weight between the nodes is shown as w_{ij} in both cases. In the neural network terminology w_{ij} is the synaptic strength between the neurons. Both, terminal node and non-terminal node outputs have interpretation as likelihood. Accordingly the weights denoted by $\boldsymbol{w_{ij}}$ in [27] are shown as the likelihood parameter $\boldsymbol{\Theta_i}$, and the output O_j of inner node *j* in [27] are shown as L_j in the following equations and figures. Let us consider a non-terminal node *j* that has two inputs, which are the outputs of two previous nodes denoted by O_1 and O_2. As the two inputs to a neuron are assumed to be independent of each other, the fuzzy memberships at the inputs can be thought to form a joint two-dimensional fuzzy membership. In this case L_j is computed by [27]

$$L_j = L_1(\theta_1)L_2(\theta_2) = e^{-\frac{\theta_1^2}{2\sigma_j^2}(O_1-1)^2}\, e^{-\frac{\theta_2^2}{2\sigma_j^2}(O_2-1)^2} \qquad (9)$$

where σ_j is a constant maximizing satisfaction of the consistency condition of possibility theory. For the two-input case $\sigma_j = 0.299$ [27]. The likelihood parameters Θ_1 and Θ_2 are selected commensurate to the amount of information they convey according to Shannon's information theorem, and they must sum up to unity for defuzzification in the rule-chaining process from node to node. Therefore the likelihood parameters in (9) are given by [27]

$$\theta_1 = \frac{1-O_1}{(1-O_1)+(1-O_2)},\ \theta_2 = \frac{1-O_2}{(1-O_1)+(1-O_2)}, \qquad (10)$$

so that (9) becomes

$$L_j = e^{-\frac{1}{2\sigma_j^2}\left(\frac{1-O_1}{(1-O_1)+(1-O_2)}\right)^2(O_1-1)^2}\, e^{-\frac{1}{2\sigma_j^2}\left(\frac{1-O_2}{(1-O_1)+(1-O_2)}\right)^2(O_2-1)^2} \qquad (11)$$

The output neuron of a fuzzy neural tree is termed as root node and the neurons connected to it as penultimate nodes. Denoting the root node's output by L_{rn} and the outputs of the penultimate nodes by $O_{p1}, O_{p2}, \ldots, O_{pn}$, then L_{rn} is obtained via the defuzzification operation

$$L_{rn} = \sum_{i=p1}^{i=pn} w_{i\_rn}\, O_i = \sum_{i=p1}^{i=pn} \theta_i O_i\ , \quad \sum_{i=p1}^{i=pn} w_{i\_rn} = \sum_{i=p1}^{i=pn} \theta_i = 1 \qquad (12)$$

The weight vector $\boldsymbol{w_{i\_rm}}$ can be aligned to the feature vector $\boldsymbol{O_i}$ writing $\boldsymbol{w_{i\_rm}} = c\boldsymbol{O_i}$, where c is a scale factor and a constant, so that the influence of a root node's input on the node's output is commensurate to the likelihood associated with the input. When we select c in such a way that the components of $\boldsymbol{w_{i\_rm}}$ sum up to unity as it should be in defuzzification, then (12) becomes [28]

$$L_{rn} = \sum_{i=p1}^{i=pn} O_i^2 \Big/ \sum_{i=p1}^{pn} O_i \qquad (13)$$

The FNT neuron yielding the perception of an object by fusing geometric and color perceptual information inputs is shown in figure 2a.

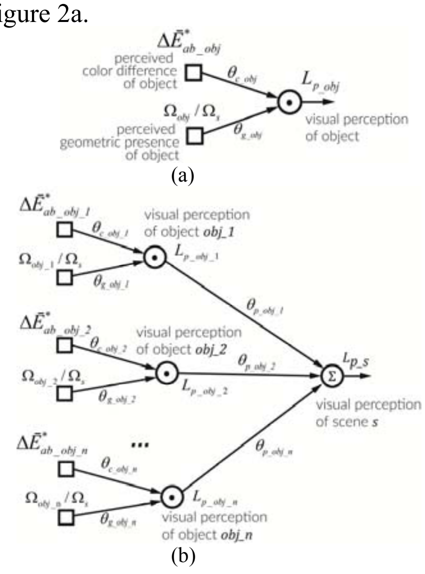


Fig. 2. Fuzzy neural tree (FNT) emulating visual perception; neurons for computing the likelihood of perception of an object (a); of a scene (b)

The normalized perceptually uniform color difference given by (6) and the geometric component of perception given by (2) are considered as fuzzy memberships at the neural tree's inputs. As the color difference and geometric inputs to visual perception neuron are assumed to be independent of each other, (8) applies, so that

$$L_{p\_obj} = L_{c\_obj}(\theta_{c\_obj}) L_{g\_obj}(\theta_{g\_obj}) \tag{14}$$

The likelihood parameters θ_c and θ_g are selected according to Shannon's information theorem, namely respectively in proportion to the amount of information each input carries [27]. Referring to (10) the likelihood parameters of the perception FNT are given by

$$\theta_{c\_obj} = \frac{1-\Delta E^*_{ab\_obj}}{(1-\Delta E^*_{ab\_obj}) + (1-\Omega_{obj}/\Omega_S)} \tag{15}$$

$$\theta_{g\_obj} = \frac{1-\Omega_{obj}/\Omega_s}{(1-\Delta E^*_{ab\_obj}) + (1-\Omega_{obj}/\Omega_S)} \tag{16}$$

Referring to (11), and using (15) and (16) in (14), the output of an inner tree node is obtained by

$$L_{p\_obj} = e^{-\frac{1}{2\sigma^2}\left[\frac{1-\Delta E^*_{ab\_obj}}{(1-\Delta E^*_{ab\_obj})+(1-\Omega_{obj}/\Omega_S)}\right]^2 (\Delta E^*_{ab\_obj}-1)^2} \; e^{-\frac{1}{2\sigma^2}\left[\frac{1-\Omega_{obj}/\Omega_S}{(1-\Delta E^*_{ab\_obj})+(1-\Omega_{obj}/\Omega_S)}\right]^2 (\Omega_{obj}/\Omega_S-1)^2} \tag{17}$$

expressing the likelihood of perceiving one of the scene objects. The standard case of scene perception is an unbiased one, i.e. there is no preference for one object over another prior to the perception act. This is accounted for by letting the vector $\boldsymbol{\theta_{p\_obj}}$ point in the same direction as likelihood vector $\boldsymbol{L_{p\_obj}}$ maximizing their alignment. Hence, referring to (13) the perception of the scene L_{p_S} is obtained via aligned defuzzification of the objects' perceptions at the FNT root node, given by

$$L_{p\_S} = f(\boldsymbol{L_{p\_obj}}) = \sum_{i=1}^{i=n} L_{p\_obj\_i}{}^2 / [\sum_{i=1}^{i=n} L_{p\_obj\_i}] \tag{18}$$

An exemplary scene subject to perception computation is shown in figure 3.

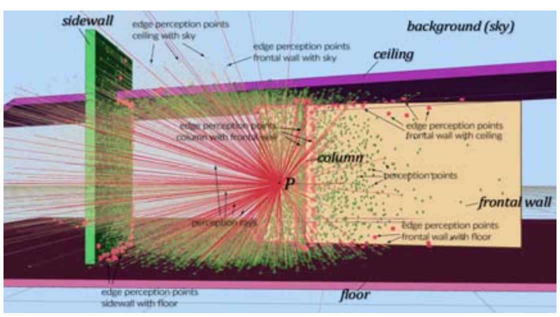
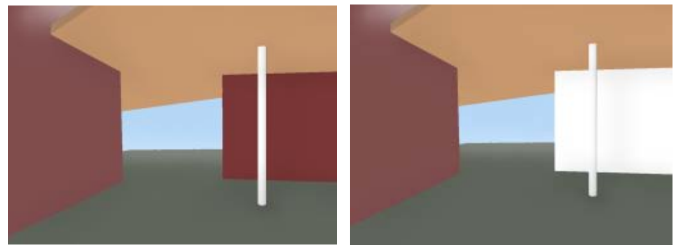


Fig. 3. An exemplary scene subject to perception computation

It consists of five building elements labeled accordingly in the figure, and a background. The intersections of the rays with the objects of the scene are shown in the figure as green dots, and they are referred to as *perception points*. Those among the perception points that simulate perception of an occlusion region are shown by red dots in the figure and referred to as *occlusion perception points*. The rendering of the same scene taken from the viewpoint labeled *P* in figure 3 with a random color composition assigned to the scene objects is shown in figure 4a. The colors assigned to the objects and the likelihoods of perception per object L_{p_obj} for the scene in figure 4a are given in table 1, together with the associated geometric perception component p_{obj} given in (2) and the color difference $\Delta\bar{E}^*_{ab\_obj}$ in (8). Figure 4b shows the same scene as figure 4a, with the difference that the frontal wall color has been changed to white, i.e. $L^*=a^*=b^*=1$. For figure 4a the scene perception is $L_{p\_S}=0.149$, whereas for figure 4b



(a) (b)

Fig. 4. Scene subject to perception with different color compositions; color composition nr. 1 (a); color composition nr. 2 (b)

the scene perception is $L_{p\_S}=0.142$ The difference as to the perceptions of the objects is seen from figure 5. From the figure, and considering the difference in the scene perceptions L_{p_S} one notes that although the perception of several objects is severely diminished when the color of the frontal wall is changed to white, the scene perception is only slightly diminishing, namely by 5%. This is due to the nature of the aligned defuzzification taking place at the root node given by (13). Reduction of the scene perception is attenuated, due to the commensurate emphasis of object remaining rather well perceived in the scene, namely the ceiling in our case.

TABLE I
PERCEPTION OF THE SCENE SHOWN IN FIGURE 4A

object	$L^*a^*b^*$	p_{obj}	$\Delta E^*_{ab\_obj}$	L_{p_obj}
frontal wall	{0.48, 0.21, 0.21}	.19	.23	.17
side wall	{0.50, 0.30, 0.30}	.09	.16	.12
ceiling	{0.78, 0.60, 0.44}	.18	.18	.15
floor	{0.37, 0.40, 0.36}	.27	.16	.17
column	{1.0, 1.0, 1.0}	.03	.28	.10

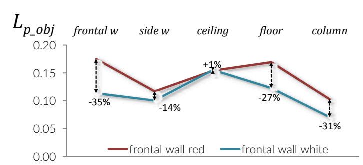


Fig. 5. Differences in object perceptions for the scenes in figure 4a and 4b

III. MULTI-OBJECTIVE NATURE OF COLOR AESTHETICS

One of the important application areas of human visual perception involving color is architectural design. The colors of the architectural objects forming an environment, such as walls, columns, floor and ceiling, have a significant influence on the visual perception of the environment and thereby on an aesthetical judgment made about the environment. The twofolded influence of perception in this circumstance is worth to emphasize. From one side, the color composition of the scene determines how intensely each object is perceived and thereby how intensely the scene as a whole is perceived. The perception of the scene clearly is precondition for a second event, namely being able to make an aesthetical judgment about the scene at all. In such judgment, color is not for the sake of the event *that* one has seen an object, but *what* one sees, i.e. what the nature of a scene is. For a scene to be judged as *aesthetical*, by definition this means that an observer should notice the exquisiteness of the scene in a sense that is intrinsic to the perception itself, e.g. [29-32].

The exquisiteness of a scene can be appreciated with the exclusive involvement of a scene's color composition as follows. A man-made environment is judged as aesthetical, when a designer has paid intelligent effort to satisfy the basic condition for an aesthetical judgment, namely the visual perception of the scene L_{p_S} in (18) can be directly realized by an unbiased observer, without intermediary abstraction. The conscious effort can be directly felt, when a certain degree of scene perception is present, while for this degree, color is involved as parsimoniously as possible per object. The low chromaticity of colors in a situation where such colors are assigned to objects arbitrarily, such as when we look at a landscape during foggy weather, then the perception of the scene is bound to have a low intensity. This is obviously fulfilling our perceptual expectation, due to the generally low color difference among weakly chromatic colors, so that it will not strike an observer as an extraordinary perception. Conversely, however, it is a rare and hence notable event, if a low chromaticity coincides with high rather high scene perception at the same time. The low probability of such an event to occur accidentally is easy to understand because each color of a scene object is a point in a large three-dimensional color space. Therefore, the combination of several objects implies a vast space of possible compositions, requiring significant conscious effort to resolve the conflict inherent to the objectives of high perception and low chromaticity.

Color has two extreme members that are technically referred to as colors, but that have unique properties among all colors. One is pitch black, the other one is pure white, which are located at the extreme ends of the lightness dimension L^* in Lab space, which is also referred to as the *achromatic* color axis. The differences among colors on the achromatic axis are exclusively due to differences in lightness L^*. Black refers to the consciously noticed absence of light stimulus in the visual spectrum, which is a theoretical boundary condition that does not exist in nature. Pure white is the perception of a photon stream containing any energy level of the visible spectrum at approximately uniform probability density per wavelength, such as sunlight. Clearly, when all objects are pure white or all are pitch black, then the scene perception is minimal, and differences in chromaticity coordinates a^* and b^* do not exist at all among objects. Introducing some chromaticity to the scene objects while remaining close to one of the achromatic extremities can be done in excessively many ways. Among these, however, only a subset of chromaticity patterns will yield highest possible scene perception. The amazement a scene is able to cause aesthetically in an observer, is that a certain intensity level of scene perception occurs despite parsimonious involvement of chromaticity and high similarity to the lightness of the achromatic reference. Although the reference lightness L^*_r used in an aesthetical judgment can theoretically be selected as any lightness level $L^*_r \in \{L^* \in \mathbb{R}: 0 \leq L^* \leq 100\}$ the conscious effort solving the scene perception-color parsimony-problem is most apparent when the reference level is either that of black or white, i.e. when $L^*_r \approx 0$, or $L^*_r \approx 100$. Then the space of finding solutions to the problem is restricted exclusively in one lightness direction; either from absence of lightness towards presence of lightness, or vice versa, but not both at the same time. This makes the aesthetical judgment more distinct, because then increase in scene perception is more likely to be due to differences in Chroma and less due to differences in lightness.

Based on the above considerations, the following verbal definitions are put forward. When a scene is a solution to the scene perception-color parsimony-problem, the scene is an aesthetical one. More specifically, when white is the reference color for an aesthetical scene, the scene should be termed as *beautiful*. Conversely when black is the reference, then the aesthetical scene should be specified as *sublime*. Ecologically this terminology can be explained as follows. The positive connotations of beauty belong to white, since in this case an aesthetical scene exposes extraordinary affinity to the light emitted from light source, such as sunlight, which is the original source of life. In case of sublimity the reference is black, since then an aesthetical scene has unusual affinity to the vastness, emptiness, and low temperature of the universe, which is the primordial threat of life. As to color, the aesthetical problem is to reach scene perception at most parsimonious involvement of color. More accurately, the color parsimony is in the sense that each object is perceived to possess minimal chromaticity. This way a color is minimally distinguishable from the achromatic reference color as to their respective chroma values. Minimal perceived chromaticity involves minimizing the color quantity known as *CIELAB chroma* denoted by C^*_{ab} [22].

$$C^*_{ab} = \sqrt{a^2 + b^2} \tag{19}$$

It is emphasized that C^*_{ab} is minimal, namely zero, for any achromatic object; including the achromatic reference colors white and black. Therefore minimizing (18) is equivalent to maximizing affinity of the object's color to the scene's aesthetical reference color with respect to chromaticity. In its normalized form $\Delta C^*_{ab\_obj}$ is given by

$$\Delta C^*_{ab\_obj} = 1 - \frac{\sqrt{a_{obj}{}^2 + b_{obj}{}^2}}{C^*_{\max}} \tag{20}$$

where C^*_{max} denotes the maximal chroma in the $L^* a^* b^*$ color space which is $C^*_{max}=181$ as to the entire visible spectrum. Due to limited RGB gamut in computer implementations, $C^*_{max}<<181$; specifically $C^*_{max}=134$ in the applications in this paper [33].

The color parsimony quantification pursued here is for comparison between different colors, as to their respective magnitude for a certain likelihood of aligned scene perception L_{p_S} in (18). Difference in lightness $L_{ab}*$ among two colors causes increased color difference ΔE_{ab} as seen from (3), and thereby influences L_{p_S}. Therefore, for determining the likelihood of chromaticity parsimony of a scene, the lightness component of an object's color L^*_{obj} should be equal to a reference lightness L^*_r. In an aesthetical judgment the reference lightness is the same as that of the achromatic reference color of the scene. That is, for a judgment concerning beauty $L^*_r \approx 100$ and for sublimity $L^*_r \approx 0$. The likelihood an object's lightness property is fulfilling this measurement condition is given by

$$\Delta L^*_{obj} = 1 - \frac{\left| L^*_r - L^* \right|}{L^*_{\max}} \tag{21}$$

Per scene object both conditions (20) and (21) are separately computed, and the likelihoods are fused by respective fuzzy neural tree AND operations at the inner nodes of the FNT in figure 6. Analog to the perception computations described in the previous section, the likelihood parameter, for the perceived chromaticity parsimony $\boldsymbol{\theta}$ is selected according to Shannon's information theorem, as described in [27]. That

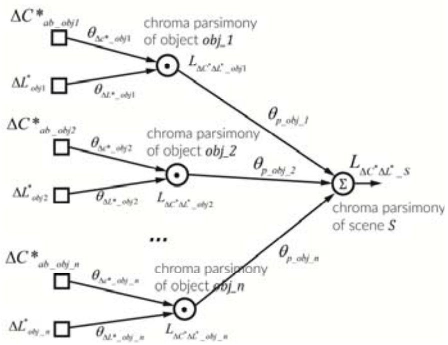


Fig. 6. Fuzzy neural tree for likelihood of CIELAB chroma parsimony

is, referring to (10) the likelihood parameters of the color parsimony FNT are given by

$$\theta_{\Delta C^*\_obj} = \frac{1 - \Delta C^*_{ab\_obj}}{(1 - \Delta C^*_{ab\_obj}) + (1 - \Delta L^*_{obj})} \tag{22}$$

$$\theta_{\Delta L^*\_obj} = \frac{1 - \Delta L^*_{obj}}{(1 - \Delta C^*_{ab\_obj}) + (1 - \Delta L^*_{obj})} \tag{23}$$

Due to (11) the output of an inner tree node is given by

$$\begin{aligned} L_{\Delta C^* \Delta L^*\_obj} &= L_{\Delta C^*_{ab}\_obj}(\theta_{\Delta C^*\_obj})\, L_{\Delta L^*\_obj}(\theta_{\Delta L^*\_obj}) \\ &= e^{-\frac{1}{2\sigma^2}\left[\frac{1-\Delta C^*_{ab\_obj}}{(1-\Delta C^*_{ab\_obj})+(1-\Delta L^*_{obj})}\right]^2 (\Delta C^*_{ab\_obj}-1)^2} \\ &\quad e^{-\frac{1}{2\sigma^2}\left[\frac{1-\Delta L^*_{obj}}{(1-\Delta C^*_{ab\_obj})+(1-\Delta L^*_{obj})}\right]^2 (\Delta L^*_{obj}-1)^2} \end{aligned} \tag{24}$$

expressing the likelihood an object fulfills the parsimony condition.

Referring to (12) the root node output $L_{\Delta C^* \Delta L^*\_S}$ is given by the defuzzification

$$L_{\Delta C^* \Delta L^*\_S} = f(\boldsymbol{L}_{\Delta C^* L^*\_obj}, \boldsymbol{\theta}_{p\_obj}) = \sum_{i=1}^{i=n} L_{\Delta C^* \Delta L^*\_obj\_i}\, \theta_{p\_obj-i} \tag{25}$$

yielding the likelihood that the scene is characterized by parsimonious chromaticity. In (25) parameter n denotes the number of scene objects. As (25) represents defuzzification in the fuzzy modeling terminology, the components of $\boldsymbol{\theta}_{p\_obj}$ must sum up to unity [27]. Hence (25) can be written as

$$L_{\Delta C^* \Delta L^*\_S} = f(\boldsymbol{L}_{\Delta C^* L^*\_obj}, \boldsymbol{L}_{\Delta p\_obj}) = \sum_{i=1}^{i=n} \left(L_{\Delta C^* \Delta L^*\_obj\_i} \frac{L_{p\_obj-i}}{\sum_{i=1}^{i=n} L_{p\_obj-i}} \right) \tag{26}$$

One notes that in (25) and (26) $\boldsymbol{\theta}_{p\_obj}$ is not aligned to $\boldsymbol{L}_{\Delta C^* L^*}$, but it is aligned to $\boldsymbol{L}_{p\_obj}$ given by (17). This alignment of the color performance vector with the likelihood of perception ensures that an object having a higher likelihood of perception will commensurately have greater influence on the perceived color character of the scene. For instance, in case an object is hardly noticeable because it occupies small vision angle and has low color difference with background, such as a minuscule stain on a piece of furniture, then irrespective of the color character the stain might possess, these hardly influence our aesthetical judgment of the scene. Conversely it is clear that when an object is highly present in our consciousness, such as a large black car in a light colored showroom, then the aesthetical character of the scene is determined greatly by the character of the car's color. In the extreme case, if one had only one object filling one's visual consciousness, such as viewing a cloudy night sky, then the aesthetical properties of the scene are exclusively due to the properties of this 'object.' It is to note that partly due to the involvement of perception in the form of the likelihood parameter $\boldsymbol{\theta}_{p\_obj}$ in the estimation of the color parsimony in (25) the judgment of the scene is an *aesthetical* one, namely the dimensions defining the aesthetical space as to color, (18) and (26) both involve perception, making both quantities dependent on the probabilistic event of a subject's private experience, and not for instance on deterministic, objective information. This is one of the corroborations of the present model with the common philosophical understanding of aesthetics. The second one is that the aesthetical judgment of color does not involve any additional perception related concept beyond the perception itself, as mentioned in the previous subsection.

Using the mathematical terms, an aesthetical judgment of a scene's color composition is measurement of the effectiveness in resolving the perceptual conflict between maximizing (26) and maximizing (18) at the same time. It is emphasized that both objectives are conflicting with each other. This is the case independent of what kind of aesthetics is pursued in the design, i.e. independent of the achromatic reference color L^*_r. The conflict can be seen as follows. The second objective's component of parsimonious chroma expressed by (20) is more satisfied for smaller values of a^* and b^*, so that the objective in (26) yields higher likelihood in this case. Low values of a^* and b^*, however, generally imply a lower degree of color difference ΔE_{ab} in (3), entailing a lower scene perception. An architectural scene is an aesthetical one, when it satisfies the condition of Pareto optimality for the two objectives given by (18) and (26) that are both subject to maximization for $L_r^* \in \{L^*: 0 \leq L^* \leq 100\}$. Explicitly, we denote the color of an object of the scene at hand by the vector $\boldsymbol{C}_{\_obj}$ given by

$$\boldsymbol{C}_{obj} = \begin{bmatrix} L^*_{obj} \\ a^*_{obj} \\ b^*_{obj} \end{bmatrix} \quad \begin{matrix} L^*_{obj} \in \{L^* : 0 \leq L^* \leq 100\} \\ a^*_{obj} \in \{a^* : -128 \leq a^* \leq 128\} \\ b^*_{obj} \in \{b^* : -128 \leq b^* \leq 128\} \end{matrix} \tag{27}$$

Each object *obj_1*, *obj_2*, …, *obj_n* of the scene has its own color. We denote a color composition of the scene by $\boldsymbol{S}$, which is given by $\boldsymbol{S}=\{\boldsymbol{C}_{obj\_1}, \boldsymbol{C}_{obj\_2}, \ldots, \boldsymbol{C}_{obj\_n}\}$. Varying components of an object's color yields another scene color composition. The totality of possible color compositions for the scene forms a set we denote by $\boldsymbol{S_t}$. A certain composition $\boldsymbol{S'}$ among $\boldsymbol{S_t}$ should be termed as *beautiful* when the following condition is valid. Given a certain color composition, in the case that there exists no other color composition that at the same time yields a greater scene perception AND greater color parsimony, while white is being used as achromatic reference color in the judgments, i.e. for $L^*_r \approx 100$, then the color composition at hand should be labeled as beautiful. $\boldsymbol{S'}$ should be termed as sublime when the same condition is valid while black is being used as achromatic reference color in the judgments, i.e. for $L^*_r \approx 0$. Regarding the perception modeling presented, it is to emphasize that the color of an object plays a role in two distinct yet related processes. Concerning the objective of maximizing the visual perception of a scene L_{ap_S}, the color of an object is not important by itself. Its relevance is exclusively in the context of yielding differences with the colors of objects occluding it and occluded by it. However, aesthetical judgment involves measurement of the chromaticity parsimony $L_{\Delta C^*\Delta L^*\_S}$, which is given as a distance to a fixed point in the color space as seen from (21), i.e. with reference to an invariant entity. Therefore one can say that in an aesthetical judgment, the color of an object plays role in both a relative and absolute sense, and it is the particularity in the involvement of both of them that determines the aesthetical performance of the scene, as well as the kind of aesthetics present, namely beauty or sublimity.

IV. Computer Experiments

Architectural design involves search for aesthetically pleasing environments. As to color, this means identification of appropriate composition $\boldsymbol{S'}$ among $\boldsymbol{S_t}$ fulfilling the sublimity or beauty conditions described above. One notes that due to combinatorial explosion $\boldsymbol{S_t}$ is a very large set, even for a moderate number of objects in a scene. Therefore, identification of beautiful or sublime color compositions by stochastic search is appropriate. In order to verify the validity of the theoretical considerations, in the following two computer experiments are carried out; one where beautiful color compositions are sought for a given scene, and one where sublime ones are sought. For this a multi-objective genetic algorithm is used, namely NSGA-II [34]. NSGA-II is a popular multi-objective genetic algorithm. Its popularity is presumably due to its minimal number of algorithm parameters, which is achieved by a parameter less technique for determining the degree of non-dominance of a solution. The technique is based on structuring the population by passing multiple surfaces through the population in the objective function space, discretizing the degree of non-domination of the population members. Due to the particularity of the Pareto ranking scheme, elitism and crowding distance computation remain also without parameters. In the experiments the algorithm parameters were selected as the following standard values, namely crossover probability *0.9*, simulated binary crossover parameter $\eta c=10$, mutation probability *0.05*, and polynomial mutation parameter $\eta m=30$. In the experiments the color of the scene background is $L^*=81.5$, $a^*=-2.5$, $b^*=-24.0$, and the colors of the scene objects are restricted by the standard RGB (sRGB) gamut [33]. The conversions of the sRGB color coordinates into *CIE* $L^*a^*b^*$ space are based on a *CIE D65 illuminant* and *2° CIE standard observer*. Figures 7a and 7b, each show a Pareto frontier of aesthetically colored scenes in objective function space. The difference between the two figures is the value of the reference lightness used during the genetic search process; in figure 7a $L^*_r=100$ and in figure 7b $L^*_r=0$. That is, the solutions in figure 7a represent beautifully colored scenes, whereas those in figure 7b represent scenes having sublime color. Four solutions among the Pareto-optimal ones are highlighted in either figure. The corresponding renderings are shown in figures 8 and 9 respectively. Metaphorically, the character of the beautiful color compositions can be said to range from gentle beauty in 8a to intense beauty in 8d. The character of the sublime color compositions can be said to range from uncanny obscurity in figure 9a to fierce discernment in 9d.

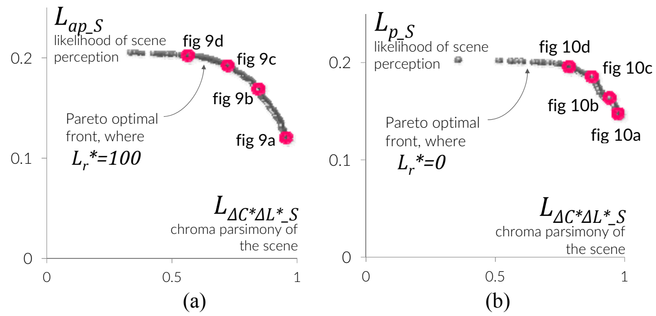


Fig. 7. Pareto front of beautiful color compositions, i.e. $L^*_r=100$ (a); of sublime color compositions, i.e. $L^*_r=100$ *(b)*

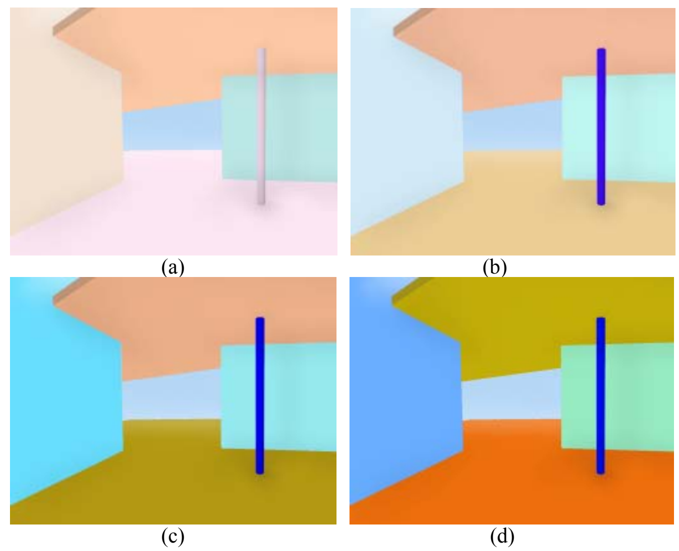


Fig. 8. The four beautiful color compositions highlighted in figure 8a

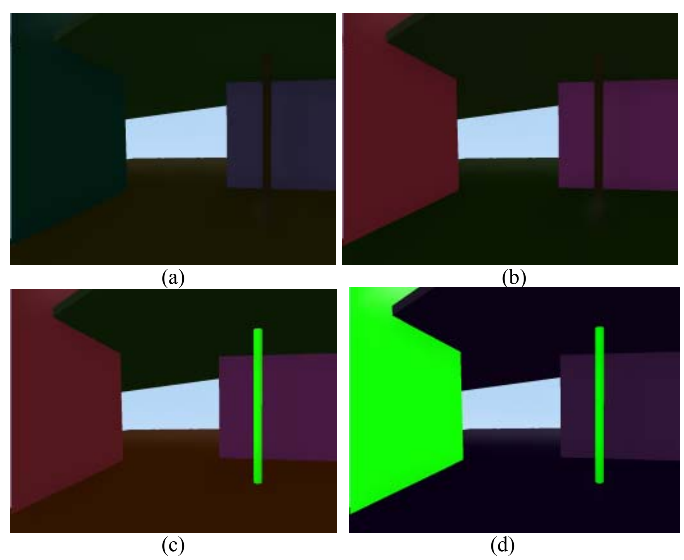


Fig. 9. The four sublime color compositions highlighted in figure 8b

CONCLUSIONS

Computer-based visual perception and aesthetical judgment of a scene's color composition are presented. Color aesthetics is identified to be the resolution of conflict that exists between two properties of a scene being perceived. The first property is that the scene should be highly perceptive, i.e. it should have some high likelihood of being perceived and hence remembered. The second property is the amount of chroma perceived in the scene should be as low as possible for a certain given intensity of scene perception. Scenes that fulfill both conditions in non-dominated manner are defined as aesthetical ones in this work. Beautiful and sublime color combinations are identified as subsets of the aesthetical ones. The definition of aesthetical designs as non-dominated solutions in a two-dimensional objective function space imply that theoretically there are infinitely many beautiful as well as sublime color compositions for a given scene geometry. Selection among them depends on designer's preference with respect to color parsimony versus intensity of scene perception. Such preference further specifies the kind of aesthetics at hand. For instance in the case of beauty, when the emphasis is on color parsimony we can term the beauty as a *gentle* kind of beauty. Conversely when scene perception is of primary interest the beauty can be characterized as an *intensive* kind. This corroborates with the common understanding of architects, that generally there exist multiple, equivalently valid solutions within the same aesthetical category. Relevant computer experiments have been set up. The validity of the theoretical considerations is verified by general acceptance. By means of the computational color perception, scenes with beautiful as well as sublime color aesthetics are established. Computational form of aesthetical judgment during a design process is a significant step for applications, where human-like visual perception and cognition are of concern. In this work color aesthetics is placed on a computational ground, reducing the imprecision in conventional aesthetical judgment, thereby contributing to the theoretical bases of disciplines that are dealing with aesthetics, such as architectural design, product design, and urbanism.

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