Architectural Design Computing Supported by Multi-Objective Optimization

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***Abstract*—Studies on computer-based perception by vision modeling are described and applied to architectural design computation supported by multi-objective optimization. The visual perception is modeled from the computational viewpoint, where the model gives a probabilistic assessment for perceiving of an object. The computational visual perception is an important step in architectural design. Further, mention may be made to general design and monitoring applications. By means of computer experiments in a virtual environment the verification of the theoretical considerations are presented, and the far reaching implications of the studies are pointed out.**

***Keywords*—visual perception; visual perception density; architecture; computational design; genetic algorithm; Pareto front**

I. INTRODUCTION

Visual perception is an important subject in several fields, including cybernetics, robotics, medicine, architecture, urbanism, and industrial design. As human gets about eighty per cent of environmental information by visual perception, it is easy to understand the importance of modelling this process, whenever human interaction with environment is subject of investigation. For establishing a model of visual perception, it is to realize that the perception should be quantified for effective treatment, such as feeding it to computer. This is because remaining in the abstract concept domain and merely dealing with perception by means of some verbal statements is trivial. The perception definition and handling approach put forward in the present research will be elaborated based on the results of the mathematical modelling in the following section. Therefore it is to note that the definition and approach presented in this work differ from existing works in some areas as such psychophysics [1-6], cognition [7-10], or image processing [11-20]. In these areas, perception as a part of brain processing is explained by some neurobiological terms rather than mathematical terms. Due to the complexity of the perception, probabilistic treatment of perception is most convenient where all the imprecisions in description of the concept are absorbed in a probabilistic model. In some object detection approaches perception is considered to be the engagement of pattern recognition. In this case Bayesian methods are appropriate [1, 21, 22]. Because of the probabilistic nature of the approach to perception, Bayesian approach is explained in more detail to emphasize the novelty of the present research, at hand.

Bayesian approach to perception is to characterize the information about the world contained in an image as a probability distribution, which combines the relative likelihoods of a viewed scene being in different states, given the available image data. The conditional probability distribution is determined in part by the image formation process, including the nature of the noise added in the image coding process, and in part by the statistical structure of the world. The Bayes's rule provides the mechanism for combining these two factors into a final calculation of the posterior distribution. This approach is based on Bayes' formula

$$p(s\,|\,i)=\frac{p(i\,|\,s)\,p(s)}{p(i)}$$

Here, s represents the visual scene, the shape and location of the viewed objects, and i represents the retinal image. $p(i|s)$ is the likelihood function for the scene, and it specifies the probability of obtaining image i from a given scene s. $p(s)$ is the prior distribution which specifies the relative probability of different scenes occurring in the world, and formally expresses the prior assumptions about the scene structure including the geometry, the lighting and the material properties. $p(i)$ can be derived from $p(i|s)$ and $p(s)$ by elementary probability theory. Namely

$$p(i)=p(i\,|\,s)\,p(s)+p(i\,|\,\bar{s})\,p(\bar{s})$$

so that Bayes' formula becomes

$$p(s\,|\,i)=\frac{p(i\,|\,s)\,p(s)}{p(i\,|\,s)\,p(s)+p(i\,|\,\bar{s})\,p(\bar{s})}$$

The posterior distribution $p(s|i)$ is a function giving the probability of the scene being s if the observed image is i. Bayesian approach is appropriate for computer vision, because for human $p(i|s)$ is almost clearly known, that is $p(i|s)=1$. Consequently, $p(i\,|\,\bar{s})=0$ and from the preceding equation

$$p(s\,|\,i)=\frac{1\times p(s)}{1\times p(s)+0\times p(\bar{s})}=1$$

which is independent of the probabilistic uncertainties about the scene. This means, as the $p(i|s)$ is definitive for human recognizing a scene, $p(s|i)$ is also definitive, being independ-

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ent of *p(s),* which is the prior assumptions about the scene structure including the geometry, the lighting and the material properties. Therefore, approaches that are based on the retinal image and ensuing image processing turn out to be trivial for human perception. In contrast to this, the effectiveness of the Bayesian approach for machine vision is due to its recursive form, providing improved estimation as the incoming information is sustained. Since the subject matter of this work is human perception, the Bayesian approaches are of minor importance.

Recapitulating the above mentioned existing perception works and associated models, it is to emphasize that the existing works address detailed aspects of visual perception, in particular in order to justify or have inspiration in the development of machine vision algorithms. However, none of the approaches addresses the uncertainty, which is a basic, common characteristic of human visual perception. The uncertainty refers to the issue that, despite the existence of a retinal image of environmental objects, this does not warrant that human becomes aware of the objects. Ignoring this basic property of human vision, the existing definitions of perception in neurobiology are insufficient as models of human vision, despite the merits of Bayesian approaches in computer vision, image processing and related fields. This becomes particularly apparent when the objects being perceived are three dimensional environments, such as real urban and architectural spaces.

The organization of the paper is as follows. Section II describes briefly the non-biological probabilistic theory for computational *visual perception*, and hereafter the computation of the *perceptual density*. Section III gives the details of computer experiments for the verification of the theory. This is followed by conclusions.

II. Probabilistic Theory for Visual Perception

A. The Basic Visual Perception Model

We start with the basics of the perception process with a simple yet a fundamental visual geometry. This is shown in figure 1, where an observer is facing and looking at a vertical plane from the point denoted by *P*. By means of looking action the observer pays visual attention equally in all directions within the scope of vision. That is, in the first instance, the observer visually experiences locations comprising the plane without any preference for one location over another concerning the direction in which the location is oriented within the scope. Each point on the plane has its own distance within the observer's scope of sight which is represented as a cone. The cone has a solid angle denoted by θ. The distance between a point on the plane and the observer is denoted by x and the distance between the observer and the plane is denoted by l_o. Since the element of visual perception $d\theta$ is determined via the associated distance, it is straightforward to proceed to express the distance of visual perception in terms of θ. From figure 1, this is given by

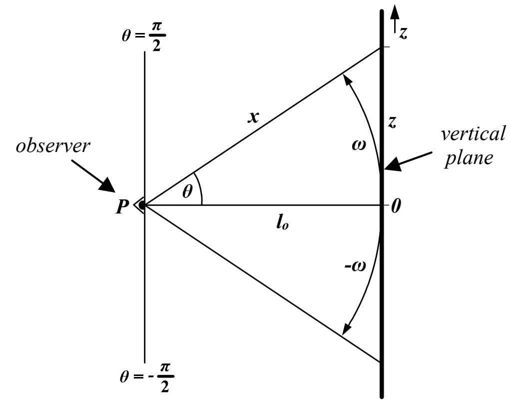

Fig. 1. The geometry of visual perception from a top view, where *P* represents the position of an observer, viewing a vertical plane at a distance l_o to the plane.

$$x = \frac{l_o}{\cos(\theta)} \quad (1)$$

Since we consider an observer who pays visual attention equally for all directions within the scope of vision, the immediate conclusion is that the probability of attention in θ direction for each θ is the same. Then the associated probability density function (pdf) $f_\theta(\theta)$ is uniformly distributed, so that $f_\theta(\theta)=1/\pi$. This axiomatic starting point ensures that there is no visual bias at the beginning of visual perception as to the differential visual resolution angle $d\theta$. Assuming the scope of sight is defined by the angle $\theta = \pm \pi/4$, the pdf f_θ is given by

$$f_\theta = \frac{1}{\pi/2} \quad (2)$$

Since θ is a random variable, the distance x in (1) is also a random variable. The pdf $f_x(x)$ of this random variable is computed and given in a previous research as

$$f_x(x) = \frac{4}{\pi} \frac{l_o}{x\sqrt{x^2 - l_o^2}} \quad (3)$$

for the interval $l_o \le x \le 2^{1/2} l_o$ [23] For this interval, the integration below becomes

$$\int_{l_o}^{\sqrt{2}\,l_o} f_x(x)\,dx = \frac{4}{\pi} \int_{l_o}^{\sqrt{2}\,l_o} \frac{l_o}{x\sqrt{x^2 - l_o^2}} = \frac{4}{\pi} \cos^{-1} \frac{l_o}{x} \Big|_{x=l_o}^{x=\sqrt{2}\,l_o} = 1 \quad (4)$$

as it should be as pdf. In this work $f_x(x)$ is defined as perception density, or alternatively we define it as *attention* in x direction. The sketch of $f_x(x)$ vs x is given in figure 2 left, and a computed plot with $l_o=1$ is shown in figure 2 right. As to (3), two observations are due. Firstly, it is interesting to note that for the plane geometry in figure 1, the visual perception is sharply concentrated close to $\theta \cong 0$ in the direction of z-axis. This striking result is in conformity with the common human experience as to visual perception. Namely, for this geometry the visual perception is strongest along the axis of the forward differential vision cone of sight, relative to the side differential vision cones. This is simply due to the fact that, for the same differential visual resolution angle $d\theta$, one can perceive visually more details on the infinite plane in the perpendicular direction. Secondly, the visual perception is given via a probability density at a point. If we consider the stimulus of perception is due to the light photons, it is the relative number of photons as stimulus at infinitesimally small interval, per unit length. Integration of these photons within a certain length gives the intensity of the stimulus, which is a measure of perception. This implies that, perception is a probabilistic concept and therefore it is different than "seeing", which is a goal oriented activity and therefore definitive. It is noteworthy to emphasize that the perception includes the brain processes to interpret an image of an object on the retina as existing object. That is, the image of an object on the retina cannot be taken for granted for the realization of that object in the brain. Normally such a realization might most likely happen, while at the same time it might not happen, too, depending on the circumstances, although the latter is unlikely to occur. The brain processes are still not exactly known so that the ability to see an object without purposely searching for it is not a definitive process but a probabilistic process and we call this process as perception. The perception is associated with a distance. This distance is designated as l_o in (3).

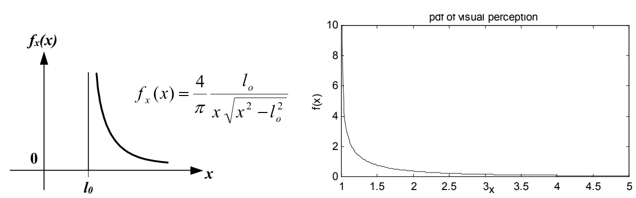

Fig. 2. Probability density function of the random variable representing the distance between eye and a location on the plane shown in figure 1; the left figure is a sketch; the right one is a plot where $l_o=1$

Also the distance along the axis z can be associated with perception. In this case the perception can be given by a different formulation [23], which is

$$f_z(z) = \frac{l_o}{\pi(l_o^2 + z^2)} \tag{5}$$

Also

$$\int_{-\infty}^{+\infty} f_z(z)\,dz = \frac{l_o}{\pi} \int_{-\infty}^{+\infty} \frac{1}{l_o^2 + z^2} = 1 \tag{6}$$

as it should be as pdf. In this work $f_z(z)$ is defined as perception density, or alternatively *attention* in z direction. The sketch of $f_z(z)$ vs z is given in figure 3 left, and a computed plot with $l_o=1$ is shown in figure 3 right. This result clearly explains the relative importance of the front view as compared to side views in human vision, as well as the influence of distance on attention.

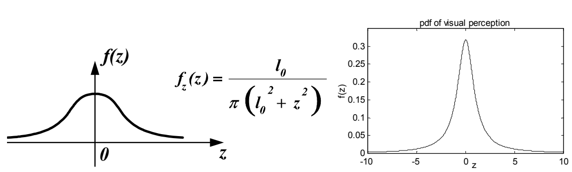

Fig. 3. Perception pdf along the z axis, i.e. parallel to the infinite plane in figure 1; the left figure is a sketch; the right one is a plot where $l_o=1$

B. *Visual Perception Model for Measurement of Perceptual Density*

Perceptual density we define as follows. It is the differential perception of the whole visual space at the point w, per unit w, where w is the location on a line, along which the whole visual perception is computed, as shown in figure 4. From the visual perception viewpoint perceptual density is a theoretical measure of density of perception. Practically it is an infinitesimally small distance giving the perceptual merits of a location measured by w per length. For any point within the semi-enclosure shown in the figure, the integral of the whole perception can be theoretically computed. For an overall perception of an interval along w, $f_w(w)$ is integrated along the interval.

Without restriction of generality, we consider the enclosure formed by three walls and by infinite plane containing the z-axis forming a convex hull. The wall dimensions are w_o, m_1, and m_2. In this geometry the whole region of interest is divided into four regions, each of which is considered separately and the results are eventually combined for the final outcome. Region I is formed by the z-axis and two broken lines denoted by $b_I(w)$ and l_o. Region II is defined by the triangle, which is formed by the broken line $b_{II}(w)$ and two walls, one with the length m_1 and the one perpendicular to it until the observation

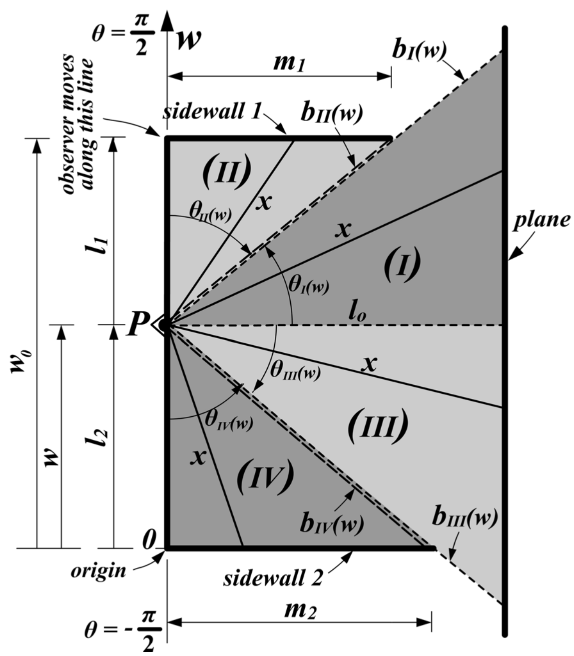

Fig. 4. Geometry involved in the computation of the probability density $f_w(w)$ as to perception along the left most wall side, where the position of perception is denoted by P.

point, shown in figure 4. The regions III and IV are defined in a similar way at the lower side of the broken line with the length of l_o.

For region I, the visual probability density is given by

$$f_{Ix}(x) = \frac{1}{\theta_I(w_o)} \frac{l_o}{x\sqrt{x^2 - l_o^2}} \tag{7}$$

,

where $\theta_I(w_o)$ is the normalization factor, which is to be determined later in the text.

To obtain the probability density with respect to the position w along the wall side, we consider the cumulative probability distribution $F(w)$ which is given by

$$F_I(w) = \int_{l_o}^{b_I(w)} f_{Ix}(x)dx = \int_{l_o}^{l_o/\cos\theta_I(w)} f_{Ix}(x)dx \tag{8}$$

where

$$b_I(w) = \frac{l_o}{\cos\theta_I(w)} = l_o\sqrt{1+tg^2\theta_I(w)} = l_o\sqrt{1+\left(\frac{w_o-w}{m_1}\right)^2} \tag{9}$$

The perceptual density $f_w(w)=p(w)$ is given by differentiating the cumulative perception distribution with respect to w. Namely,

$$f_w(w) = \frac{dP_T(w)}{dw} = p(w) \tag{10}$$

where $P_T(w)$ is the cumulative total perception normalized to unity, so that $P_T(w_o)=1$, which means the cumulative perception of the convex hull, namely the cumulative perception of the whole closed space is unity. Also, we can write

$$\int_0^w f_w(w)dw = \int_0^{P_T(w)} dP_T(w) = P_T(w) \tag{11}$$

and in particular for $w=w_o$ then

$$\int_0^{w_o} f_w(w)dw = \int_0^{P_T(w_o)} dP_T(w) = P_T(w_o) = 1 \tag{12}$$

The cumulative probability distribution $P_T(w)$ is the integral of perceptual density along the line w until the point w. It is a monotone increasing function and always positive as the perception is always positive, by definition.

The perceptual density $f_w(w)=p(w)$ is computed for each region separately, and they are combined as a composite perceptual density valid for the whole range of w. The computation of perceptual density $f_w(w)=p(w)$ is carried out as follows. The differentiation of $F_I(w)$ in (8) with respect to w is carried out according to Leibniz integral rule. The Leibniz integral rule gives a formula for differentiation of a definite integral whose limits are functions of the differential variable. It is sometimes known as *differentiation under the integral sign*.

$$\frac{\partial}{\partial w}\int_{a(w)}^{b(w)} f(x,w)dx = \int_{a(w)}^{b(w)} \frac{\partial f(x,w)}{\partial w}dx + f(b(w),w)\frac{\partial b(w)}{\partial w} - f(a(w),w)\frac{\partial a(w)}{\partial w} \tag{13}$$

By the application of Leibniz rule for (7), we obtain

$$\frac{\partial}{\partial w}\int_{a(w)}^{b_I(w)} f_{Ix}(x)dx = \int_{a(w)}^{b_I(w)} \frac{\partial f_{Ix}(x)}{\partial w}dx + f_{Ix}(b(w))\frac{\partial b(w)}{\partial w} - f_{Ix}(a(w))\frac{\partial a(w)}{\partial w} \tag{14}$$

where $a(w)=l_o$ and constant, so that (14) boils down

$$f_{Iw}(w) = f_I(b_I(w))\frac{\partial b}{\partial w}. \tag{15}$$

In (15) $b_I(w)$ is given by (9), so that

$$f_{Iw}(b(w)) = \frac{1}{\theta_I(w_o)}\frac{1}{\sqrt{1+\left(\frac{w_o-w}{m_1}\right)^2}}\;\frac{1}{l_o\left(\frac{w_o-w}{m_1}\right)} \tag{16}$$

$$\frac{\partial b(w)}{\partial w} = \frac{l_o}{m_1}\frac{\left(\frac{w_o-w}{m_1}\right)}{\sqrt{1+\left(\frac{w_o-w}{m_1}\right)^2}} \tag{17}$$

Substitution of (16) and (17) into (15) yields

$$f_{Iw}(w) = \frac{1}{\theta_I(w_o)}\frac{1}{m_1}\frac{1}{1+\left(\frac{w_o-w}{m_1^2}\right)^2} = \frac{1}{\theta_I(w_o)}.\frac{m_1}{m_1^2+(w_o-w)^2} \tag{18}$$

This is the probability density of visual perception with respect to w for the *region I*. It is interesting to note that this is not dependent on the distance l_o.

To determine the normalization factor $\theta_I(w_o)$, from (18) we write

$$F_{Iw}(w) = \frac{1}{\theta_I(w_o)}\int_{w_o}^{0}\frac{m_1}{m_1^2+(w_o-w)^2}dw = \frac{1}{\theta_I(w_o)}arctg\frac{w}{m_1} \tag{19}$$

Since for $w=w_o$, $F_{Iw}(w)=1$, from (19) it follows that

$$\theta_I(w_o) = arctg\frac{w_o}{m_1} \tag{20}$$

For the region *II* the computation is carried out by Leibniz rule with $l_I=w_0-w$. The reason for two different methods for the same calculation is for the purpose of giving insight into the perception concept introduced in this paper rather than providing only some abstract mathematical calculations omitting the perception concept, which is central in this research. In this context, these two methods can be categorized as follows. *Leibniz rule with* $l_I=w_0-w$, is a purely abstract mathematical method leading to the result and the *method of function of variable* is mathematical method where perception related probabilistic concept is central. We start with the probability density given by

$$f_{IIw}(x,w)=\frac{1}{\theta_{II}(w_o)}\frac{(w_o-w)}{x\sqrt{x^2-(w_o-w)^2}} \quad (21)$$

In (21), the pdf is dependent on the distance w_o-w from the leftmost wall side. Since we are interested in the probability density with respect to x, in (18) w is a parameter which can be considered constant until the cumulative probability of $f_{IIw}(x,w)$ with respect to x is calculated.

To obtain the pdf of the position w along the wall side with respect to the region II, we consider the cumulative probability $F_{II}(w)$ which is given by

$$F_{II}(w)=\int_{w_o-w}^{b_{II}(w)} f_{IIx}(x,w)dx \quad (22)$$

where

$$b_{II}(w)=\frac{(w_o-w)}{\cos\theta_{II}(w)}=(w_o-w)\sqrt{1+\left(\frac{m_1}{w_o-w}\right)^2} =\sqrt{(w_o-w)^2+m_1^2} \quad (23)$$

so that, the probability distribution $F_{II}(w)$ is given by

$$F_{II}(w)=\int_{w_o-w}^{\sqrt{(w_o-w)^2+m_1^2}} f_{IIx}(x,w)dx \quad (24)$$

The Leibniz integral rule given by (13) becomes

$$\frac{\partial}{\partial w}\int_{a(w)}^{b_{II}(w)} f(x,w)dx=\int_{a(w)}^{b_{II}(w)}\frac{\partial f(x,w)}{\partial w}dx+ f(b_{II}(w),w)\frac{\partial b_{II}(w)}{\partial w}-f(a(w),w)\frac{\partial a(w)}{\partial w} \quad (25)$$

where $a(w)=w_o-w$. In (25) the first term at the right hand side gives

$$\begin{aligned}(f_{IIx})_1&=\frac{1}{\theta_{II}(w_o)}\int_{w_o-w}^{b_{II}(w)}\frac{\partial f_{IIx}(x,w)}{\partial w}dx\\ &=\frac{1}{\theta_{II}(w_o)}\int_{w_o-w}^{b_{II}(w)}\frac{\partial\left[x\sqrt{\frac{x^2}{(w_o-w)^2}-1}\right]^{-1}}{\partial w}dx\\ &=\frac{1}{\theta_{II}(w_o)}\int_{w_o-w}^{b_{II}(w)}\frac{x}{\left[x^2-(w_o-w)^2\right]^{-\frac{3}{2}}}dx\\ &=\frac{1}{\theta_{II}(w_o)}\frac{1}{\sqrt{x^2-(w_o-w)^2}}\Bigg|_{w_o-w}^{b_{II}(w)}\\ &=\frac{1}{\theta_{II}(w_o)}\left[\frac{1}{m_1}-\frac{1}{\sqrt{x^2-(w_o-w)^2}}\Bigg|_{x=w_o-w}\right]\end{aligned} \quad (26)$$

The second term at the right hand-side in (25) is computed as follows

$$f_{IIx}(b_{II}(w))\frac{\partial b_{II}(w)}{\partial w}, \quad (27)$$

where $b_{II}(w)$ is given by (23). Considering that

$$f_{IIx}(x)=\frac{1}{\theta_{II}(w_o)}\frac{w_o-w}{x\sqrt{x^2-(w_o-w)^2}} \quad (28)$$

we write

$$f_{IIx}(b_{II}(w),w)=\frac{1}{\theta_{II}(w_o)}\frac{1}{m_1}\frac{w_o-w}{\sqrt{(w_o-w)^2+m_1^2}} \quad (29)$$

$$\frac{\partial b_{II}(w)}{\partial w}=-\frac{w_o-w}{\sqrt{(w_o-w)^2+m_1^2}} \quad (30)$$

Substitution of (29) and (30) into (27) yields

$$(f_{IIx})_2=-\frac{1}{\theta_{II}(w_o)}\frac{1}{m_1}\frac{(w_o-w)^2}{(w_o-w)^2+m_1^2} \quad (31)$$

The third term at the right hand-side in (25) is computed as follows

$$(f_{IIx})_3=f_{IIx}(w_o-w,w)\frac{\partial(w_o-w)}{\partial w} \quad (32)$$

which gives

$$(f_{IIx})_3=-\frac{1}{\theta_{II}(w_o)}\frac{w_o-w}{x\sqrt{x^2-(w_o-w)^2}}\Bigg|_{x=w_o-w} \quad (33)$$

and finally

$$(f_{IIx})_3=-\frac{1}{\theta_{II}(w_o)}\frac{1}{\sqrt{x^2-(w_o-w)^2}}\Bigg|_{x=w_o-w} \quad (34)$$

The summation of (26) (31) and (34) yields

$$\begin{aligned}f_{IIw}(w)&=\frac{1}{\theta_{II}(w_o)}\left[\frac{1}{m_1}-\frac{1}{\sqrt{x^2-(w_o-w)^2}}\Bigg|_{x=w_o-w}\right]-\\ &\frac{1}{\theta_{II}(w_o)}\frac{1}{m_1}\frac{(w_o-w)^2}{(w_o-w)^2+m_1^2}+\\ &\frac{1}{\theta_{II}(w_o)}\frac{1}{\sqrt{x^2-(w_o-w)^2}}\Bigg|_{x=w_o-w}\\ &=\frac{1}{m_1}\left[1-\frac{(w_o-w)^2}{(w_o-w)^2+m_1^2}\right]\end{aligned} \quad (35)$$

and finally

$$f_{IIw}(w)=\frac{1}{\theta_{II}(w_o)}\frac{m_1}{(w_o-w)^2+m_1^2} \quad (36)$$

This is the probability density of visual perception for the region II.

The normalization factor $\theta_{II}(w_o)$ is obtained following the same procedure as was done for region I. Then it is obtained to be

$$\theta_{II}(w_o)=arctg\frac{w_o}{m_1} \quad (37)$$

Alternatively this result can be obtained explicitly in the following way.

The normalization factor $\theta_{II}(w_o)$ is obtained following the same procedure as was done for region I. Namely the cumulative probability is given by

$$\begin{aligned}F_{II\,w}(w)&=\frac{1}{\theta_{II}(w_o)}\int_{w_o-w}^{w_o}\frac{m_1}{(w_o-w)^2+m_1^2}dw\\ &=\frac{1}{\theta_{II}(w_o)}arctg\frac{w}{m_1}\end{aligned} \quad (38)$$

Since for $w=w_o$, $F_{IIw}(w_o)=1,$ from (37) it follows that

$$\theta_{II}(w_o) = arctg\frac{w_o}{m_1} \tag{39}$$

Following the same strategy as it was done for regions *I* and *II*, in the same way, $f_{III\,w}(w)$ and $f_{IV\,w}(w)$ can be computed as

$$f_{III\,w}(w) = \frac{1}{\theta_{III}(w_o)}.\frac{m_2}{m_2^2 + w^2} \tag{40}$$

$$f_{IV\,w}(w) = \frac{1}{\theta_{IV}(w_o)}\frac{m_2}{m_2^{\,2} + w^2} \tag{41}$$

For $F_{III}(w)$ and $F_{IV}(w)$ using (40) and (41), we obtain

$$F_{III}(w) = \frac{1}{\theta_{III}(w_o)}\int_0^w \frac{m_2}{m_2^2 + w^2}\,dw = \frac{1}{\theta_{III}(w_o)} arctg\frac{w}{m_2} \tag{42}$$

and

$$F_{IV}(w) = \frac{1}{\theta_{IV}(w_o)}\int_0^w \frac{m_2}{m_2^{\,2} + w^2}\,dw = \frac{1}{\theta_{IV}(w_o)} arctg\frac{w}{m_2} \tag{43}$$

The normalization coefficients $\theta_{III}(w_o)$ and $\theta_{IV}(w_o)$ are found to be

$$\theta_{III}(w_o) = \theta_{IV}(w_o) = arctg\frac{w_o}{m_2} \tag{44}$$

Since the events in regions *I, II, III,* and *IV* are mutually exclusive, the total probability density is

$$f_w(w) = \frac{\theta_I(w_o)f_{I\,w}(w) + \theta_{II}(w_o)f_{II\,w}(w)}{\theta_I(w_o)+\theta_{II}(w_o)+\theta_{III}(w_o)+\theta_{IV}(w_o)} + \frac{\theta_{III}(w_o)f_{III\,w}(w) + \theta_{IV}(w_o)f_{IV\,w}(w)}{\theta_I(w_o)+\theta_{II}(w_o)+\theta_{III}(w_o)+\theta_{IV}(w_o)} \tag{45}$$

The substitution of $\theta_I(w_o), \theta_{II}(w_o), \theta_{III}(w_o), \theta_{IV}(w_o)$ from (20), (39) and (44), into (45) yields the probability density as

$$f_w(w) = \frac{1}{arctg\frac{w_o}{m_1} + arctg\frac{w_o}{m_2}}\left[\frac{m_1}{m_1^2 + (w_o - w)^2} + \frac{m_2}{m_2^2 + w^2}\right] \tag{46}$$

It is to note that

$$\int_0^{w_o} f_w(w)\,dw = 1 \tag{47}$$

as it should verify as pdf.

The determination of $f_w(w)$ can be carried out at any place of the area delimited by the walls m_1 , m_2 , w and the plane shown in figure 4, by simply passing a line parallel wall *w,* where the line has a smaller l_o compared to the position of *w*. In this case only the parameters playing role on the probability density computation change, namely the length of the side walls m_1 and m_2. In other words new probability density formulation is not necessary, while only the same computer experiments are repeated with the new parameters.

III. Computational Design

As a computational design experiment, the plot of the perceptual density $f_w(w)$ given by (46) for a geometry with the parameters $w_0=5m$, $m_1=2.5m$, $m_2=2.5m$ is shown in figure 5 upper. The experimental setup of this computer experiment is shown in a plan view in virtual reality in the same figure below the plot of the pdf. For simplicity, in this experimental setup and the ensuing ones, only a single observation point is shown in the plan view, namely at $w=w_o/2$. The function has its minimum at $w=w_o/2$, and two local maxima near the side walls.

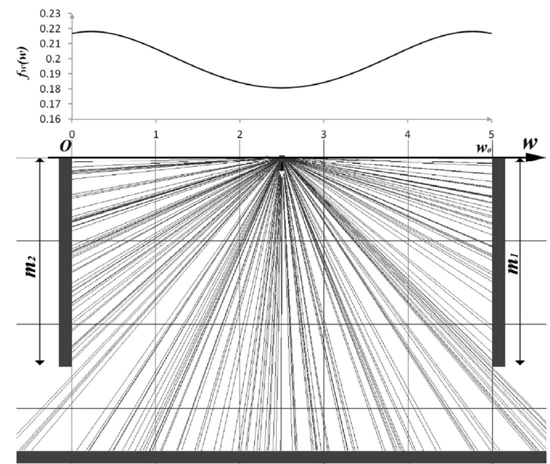

Fig. 5. Plot of $f_w(w)$, where $w_0=5m$, $m_1=2.5m$, $m_2=2.5m$ (upper), and corresponding geometry (lower).

An experiment similar to that shown in figure 5 is carried out with the setting $w_o=2m$, $m_1=2.5m$, $m_2=1.0m$. This yields the probability density $f_w(w)$ as shown in figure 6 upper.

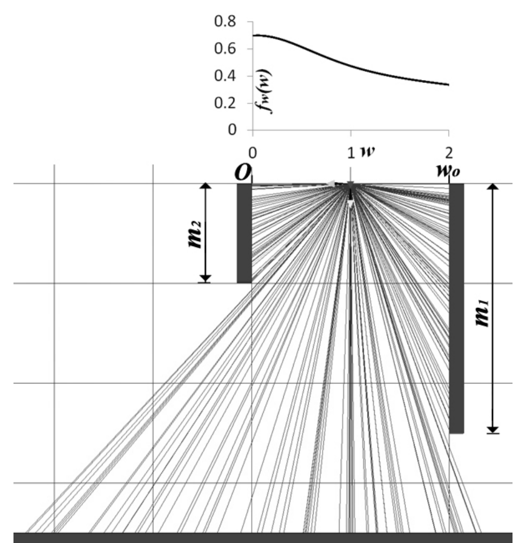

Fig. 6. Plot of $f_w(w)$ where $w_0=2m$, $m_1=2.5m$, $m_2=1m$ (upper), and the corresponding geometry (lower).

The corresponding experimental setup is shown below the pdf in the same figure. Considering the shape of the probability density $f_w(w)$ for the different geometries in figure 5 and 6 one notes that for asymmetrical spaces, i.e. geometries where m_1 and m_2 are different, the shape of $f_w(w)$ becomes asymmetric, and the higher values of the function occur near the extremity belonging to the shorter wall.

In an architectural design, given the width of a room denoted by w_o in figures 5 and 6, the task is to determine the wall lengths m_1 and m_2, as well as the viewing location w, so that panoramic as well as perceptual density objectives are simultaneously satisfied. A panoramic view is considered as the angle at the place observer stands with an arc with respect to the wall lengths m_1, and m_2 as shown in figure 7. The panoramic view angle θ is maximal for a pair of wall lengths m_1, m_2 and the angle subtended by the arc m_1m_2 shown by a blue line in figure 7. Such an angle is defined by a circle passing from the endpoints of the walls *E1, E2* and it is tangent to the w axis, as seen in the figure. The tangent point is the most favorable panoramic point, and designated as w_T in the figure. The center of such a circle is found by taking the w-axis as directrix and the endpoints *E1, E2* focal points of two parabolas which intersect. The intersection point is the center of the circle. The design optimization is carried out in the following way. We define two objectives. The first one is maximizing $f_w(w)$. The second one is minimizing the distance between w_T and w. These are common goals in an architectural design, where a person entering a space should experience a large panorama, while at the same he should become highly aware of the spatial enclosure. The optimization involves the constraint $0<w_t<w_o$. The parameter value for w_o is set to $w_o=5$.

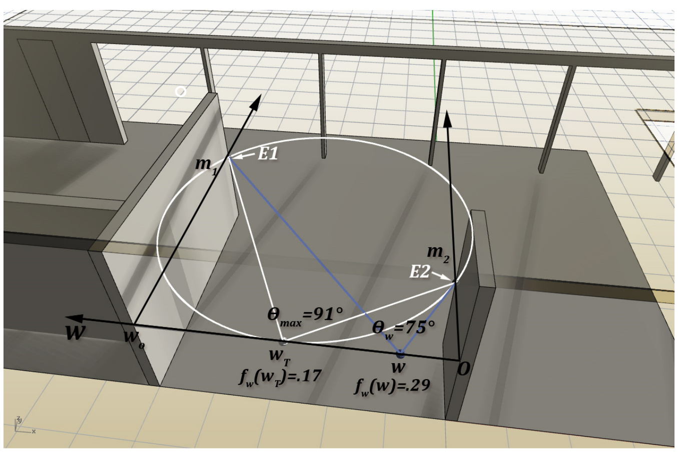

Fig. 7. One of the Pareto optimal designs, marked by an arrow in figure 8, where $f_w(w)=0.29/m$ and $|w_T\text{-}w|=1.64m$

The optimization is carried out using *NSGA-II* with a population size of 300. NSGA-II is a well-known multi-objective genetic algorithm. Its popularity is presumably due to its minimal number of algorithm parameters, which is achieved through a parameter less technique determining the degree of non-dominance of a solution in the form of rank given by an integer. The algorithm parameters were selected as the following standard values, namely crossover probability *0.9*, simulated binary crossover parameter $\eta_c=10$, mutation probability *0.05*, and polynomial mutation parameter $\eta_m=30$. The resulting Pareto front is shown in figure 8. One notes that the objectives are $-f_w(w)$ and $|w_T\text{-}w|$ in the figure, so that both functions are represented as subject to minimization. One of the Pareto solutions is selected for implementation in an architectural design. The solution is illustrated in figure 7 and the same solution is marked in figure 8 by an arrow. For this solution, the perceptual density $f_1=f_w(w)=0.29/m$ and $f_2=|w_T\text{-}w|=1.64m$. The design has the following values for the decision variables: $m_1=3.91m$, $m_2=1.54m$, $w=0.84m$. For this design $\Theta_w=75°$, and the location of maximal angle Θ is $w_T=2.48m$, where $f_w(w)=0.17/m$, $\Theta_{max}=91°$.

The solution in figure 7 satisfies the architectural demand for high perceptual density and high panoramic angle at the same time. Pareto front offers flexibility as to selecting non-dominated perceptual density and panorama at the same time.

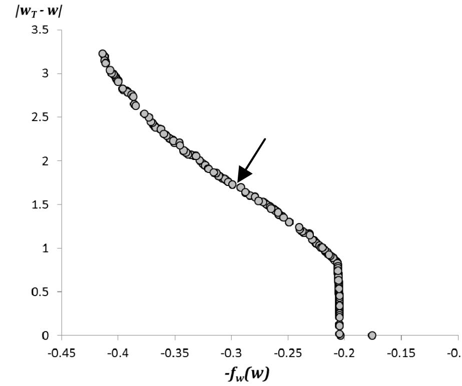

Fig. 8. Pareto front, where $|w_T\text{-}w|$ and $-f_w(w)$ are subject to minimization; the solution shown in figure 7 is marked by an arrow

IV. Discussion

From the architectural design viewpoint, the usage of multi-objective evolutionary algorithm for satisfaction of perceptual demands is an important step. The importance is due to the fact that visual perception is quintessential concern in architectural design, while it is difficult to handle computationally. The difficulty is due to the probabilistic nature of perception causing non-linearity in the relation between variables defining the geometry of objects being viewed, and perception events occurring in the mind. Therefore optimization of perceptual properties generally is to be carried out by means of stochastic search algorithms. The importance of visual perception in Architecture and architectural design is clarified as follows. When we compare a building that is to be classified as Architecture with a building that is not to be classified as Architecture, then the former will be distinct from the latter due to the involvement of visual perception considerations during the design of the former. That is, Architecture possesses desirable visual perception properties. A basic experience is to notice the existence of an object. In this case the property of a building component to be determined is the probability the object is seen, i.e. the degree by which it enters the awareness of an observer, and the probability quantifies the perception of the object. In case the object concerned is the entire space surrounding the observer, then seeing the object is certain, and this is valid for any viewpoint within the spatial enclosure making them equivalent in this respect. However, different viewpoints within the same space are not equivalent when compared with respect to their respective perceptual density, which is a perceptual property that is particularly important in architecture. This is because architectural designs are composition of spaces, where the individual building elements, such as walls, serve the purpose of defining the space, while they may be less significant as individual objects of perception. Despite the importance of perceptual density concept in Architecture, it is a subtle property to experience by an architect due to complexity, and therefore the property could hardly be quantified with precision. The computational perception and design approach presented in this paper permits accurate treatment of this property in design. Multi-objective optimization yielding solutions with desirable perceptual properties is an example of the value that computations have in architectural design.

V. Conclusion

Visual Perception is modeled from the computational viewpoint, where the model gives a probabilistic assessment for perceiving of an object. Based on the perception computations, architectural design having desirable perceptual properties is attained by multi-objective evolutionary algorithm. The perception computations presented uniquely treat the uncertain nature of visual perception that human commonly experiences. Despite the existence of a retinal image of environmental objects, this does not warrant that human becomes aware of the objects. The uncertainty characterizing perception is sometimes referred to as ‘overlook,’ ‘deficient remembrance’, and so on, and despite its prevalence it remained without computational treatment in the human perception literature. Through the presented computational perception approach, the existing works on visual perception are desirably complemented by unambiguous conceptualization and modeling. Also, from visual perception, the concept of perceptual density, as a derivative of perception, is introduced. The density quantifies how perception of an entire spatial enclosure differs along a line in the space, which is an important issue in architectural design. The validity of the concepts is verified by means of computer experiments in virtual reality. In computational architectural design, perceptual density is an objective that can be in conflict with other perceptual requirements, such as maximal panoramic view. In a computational design experiment this is investigated via Pareto front, obtained by a constrained multi-objective genetic algorithm and presented. The results reveal that for a high perception density and high perception of panorama at the same time, the entrance to the space in question must be situated at location between the middle point of the sidewalls and the shorter wall. This architectural knowledge is not to be obtained, without invoking the probabilistic and evolutionary computations described which is due to subtlety of the dependence of perception on viewing position. Architectural design computation supported by multi-objective optimization is a unique approach to handle the delicate, yet decisive differences among architectural designs with effectiveness. As the quality of an architectural design is determined by its perceptual properties, the work exemplifies the significance that computations have in this domain.

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