A Fuzzy Neural Tree for Possibilistic Reliability

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***Abstract* - An innovative neural fuzzy system is considered for possibilistic reliability using a neural tree structure with nodes of neuronal type. The total tree structure works effectively as a fuzzy logic system where the possibility theory plays important role with Gaussian possibility distribution at the nodes. The structure of the tree is determined by domain knowledge and each node represents a component of the system of concern. The reliabilities of the nodes are dependent on the reliabilities of the preceding nodes. The relationships among the nodes form the core of possibilistic reliability. For each input reliability composition the status of the system is known and interpreted not only at the system output but also at the granulated level at the system sub-domains which are represented by node outputs. The research is described in detail and a demonstrative computer experiment is reported.**

I. Introduction

Reliability theory is a cross disciplinary engineering discipline. Reliability is a measure of the expected performance of an engineering product to operate without failures under specific conditions for a given period of time. The measure is, in broad sense, is probabilistic. The classical reliability theory is based on two basic assumptions.

1-At any given time, the device is either functioning state or in failed state

2-The behaviour of the device between two critical states can adequately be characterized in terms of probability theory.

These assumptions make sense only when the events are mathematically ideal or crisp, large collection of data are available, and observed frequencies are indicators of the underlying physical process. Unfortunately these assumptions are in most cases not valid but they are taken granted for a reasonable approximation to a true reliability. Especially, for critical engineering systems where events are not acceptable, like Chernobyle nuclear accident in 1986, we cannot have enough data to make accurate probabilistic assessments and the complexity of the event does not allow defining a universe of discourse where all the probabilistic outcomes are defined. Therefore, especially in such engineering systems although the reliability assessments are extremely of concern, unfortunately, in such very situations the reliability data are, not coincidently, poor. To deal with the issue, soft computing can be of great help to make some key assessments which can be used for further reliability computations. Having this in mind, the work is motivated by adapting the work on fuzzy neural tree for soft computing developed earlier for intelligent design studies in design [1, 2], in order to make reliability assessment in a soft computing framework using the possibility theory [3-6] founded by Zadeh [7]. The presentation of this work is organized as follows. Section II describes the neural-tree-fuzzy system developed in this research as a base for the possibilistic reliability system being developed. Section III describes the fuzzy neural tree in the possibilistic reliability framework. Section IV gives a computer experiment example which is followed by discussion and conclusions.

II. Neural tree fuzzy model

Neural tree networks are in the paradigm of neural networks with marked similarities in their structures. A neural tree is composed of terminal nodes, non-terminal nodes, and weights of connection links between the pairs of nodes. The non-terminal nodes represent neural units and the neuron type is an element introducing a non-linearity simulating a neuronal activity. In the present case, this element is a Gaussian function which has several desirable features for the goals of the present study; namely, it is a radial basis function ensuring a solution and the smoothness. At the same time it plays the role of possibility distribution in the tree structure which is considered to be a fuzzy logic system as its outcome is based on fuzzy logic operations thereby associated reasoning. An instance of a neural tree is shown in Fig. 1.

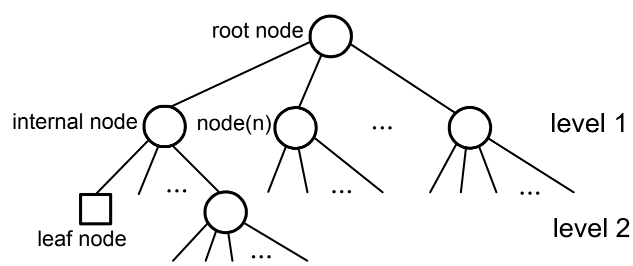

Fig. 1. The structure of a neural tree

Each terminal node, also called *leaf*, is labelled with an element from the terminal set $T=\{x_1, x_2, ..., x_n\}$, where x_i is the i-th component of the external input $\boldsymbol{x}$ which is a vector. Each link (j,i) represents a directed connection from node j to node i. A value w_{ij} is associated with each link. In a neural tree, the root node is an output unit and the terminal nodes are input units. The node outputs are computed in the same way as computed in a feed-forward neural network. In this way, neural trees can represent a broad class of feed-forward networks that have irregular connectivity and non-strictly layered structures. In particular, in the present work the nodes are similar to those used in a radial basis functions network with the Gaussian basis functions.

In the neural tree considered in this work, the output of i-th terminal node is denoted w_i and it is introduced to a non-terminal node. A non-terminal node consists of a Gaussian radial basis function.

$$f(X) = w\phi(\| X - c \|^2) \tag{1}$$

where $\phi(.)$ is the Gaussian basis function, $\boldsymbol{c}$ is the centre of the basis function. The Gaussian is of particular interest and used in this research due to its relevance to fuzzy-logic. The width of the basis function σ is used to measure the uncertainty associated with the node inputs designated as external input X. The output of i-th terminal node w_i is related to X by the relation

$$X_i = w_i w_{ij} \tag{2}$$

where w_{ij} is the weight connecting terminal node i to terminal node j. It connects the output of a basis function to a node in the form of an external input. This is shown in Fig. 2.

The centres of the basis functions are the same as the input weights of that node. Therefore, for a *terminal node connected to a non-terminal node*, we can express the non-terminal node output denoted by O_j, as

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{X - w_{ij}}{\sigma_j}\right]^2\right) \tag{3}$$

which becomes due to (2)

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{w_{ij}(w_i - 1)}{\sigma_j}\right]^2\right) \tag{4}$$

where j is the layer number; i denotes the i-th input to the node; w_i is the degree of possibility at the output of the terminal node; w_{ij} is the weight associated with the i-th terminal node and the non-terminal node j.

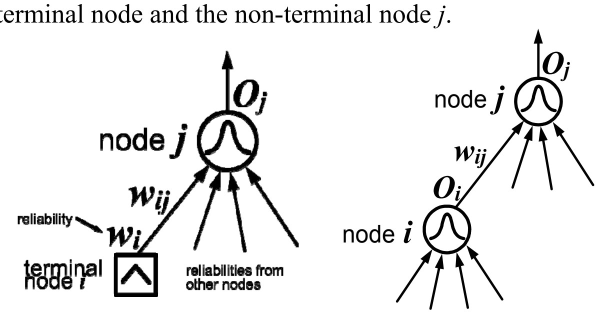

Fig. 2. The detailed structure of a neural tree with respect to different type of node connections

For a *non-terminal node connected to a non-terminal node*, (3) becomes

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{w_{ij}O_i - w_{ij}}{\sigma_j}\right]^2\right) \tag{5}$$

and further

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{w_{ij}(O_i - 1)}{\sigma_j}\right]^2\right) \tag{6}$$

We can express (4) and (6) in the following form, respectively.

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{(w_i - 1)}{\sigma_{wj}}\right]^2\right) \tag{7}$$

and

$$O_j = \exp\left(-\frac{1}{2}\sum_i^n \left[\frac{(O_i - 1)}{\sigma_{wj}}\right]^2\right) \tag{8}$$

where

$$\sigma_{wj} = \frac{\sigma_j}{w_{ij}} \tag{9}$$

Referring to (6)-(8), the following observations are essential.

- for each node, parsimoniously, a common width is defined in place of several ones each of which for a term in the summation.
- Centre of a Gaussian is always equal to unity.
- Width of a Gaussian is scaled by the input weight w_{ij} as seen in (8). In other words, as to width, the shape of Gaussian fuzzy possibility distribution is dependent on the input weights w_{ij} of the respective Gaussian. This has important implication; namely the Gaussiam radial basis function has multi-dimension and at each dimension the Gaussian width is different and commensurate with the respective weight w_{ij}. It is commensurate in the sense that the smaller the weight the larger the respective Gaussian width thereby smaller effect at the Gaussian output of that input.
- Derivative w.r.t. w_i for the terminal nodes and w.r.t. O_i for non-terminal nodes is always positive. Namely, for a terminal node, from (7)

$$\frac{\partial O_j}{\partial w_i} = -\frac{w_{ij}^2}{\sigma_j^2}(w_i - 1)O_j \tag{10}$$

Since $0 < w_i \leq 1$ and O_j is positive, it follows that

$$\frac{\partial O_j}{\partial w_i} \geq 0 \tag{11}$$

In the same way, for a non-terminal node, from (8)

$$\frac{\partial O_j}{\partial O_i} = -\frac{w_{ij}^2}{\sigma_j^2}(O_i - 1)O_j \tag{12}$$

Since $\leq O_i \leq 1$ and O_j is positive, it follows that

$$\frac{\partial O_j}{\partial O_i} \geq 0 \tag{13}$$

Above, (11) and (13) are important conclusions; namely, the neural tree output is an increasing function of the inputs. This means with the increasing values of the input, the tree output is always increasing and vice versa. This implies that, in this structure, only the left side of the Gaussian possibility distributions of the non-terminal nodes are used. This is shown in figure 3. Therefore in the figure the right side of the Gaussian is shown with a broken line. Also it is worth to mention that the normalisation of the possibility distribution is made at the location 1 and therefore it is characterized only by its width as the shape and location is known. As it will be seen later, the width is determined by training.

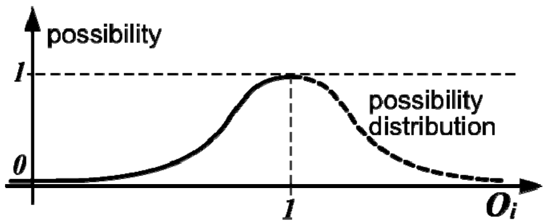

Fig. 3. Fuzzy possibility distribution at a non-terminal node.

III. Fuzzy Neural Tree in a Possibilistic Reliability Framework

In the possibilistic reliability, the fuzzy neural tree weights are determined according to failure modes and effects analysis (FMEA) [8] and in this capacity they represent the affinity of one node to another connected. With other words in contrast to the case in neural networks, here the weights w_{ij} are determined by domain knowledge whereas in neural networks they are determined by training. At *i-th* terminal node the reliability of the associated system component is shown as w_i. At a non-terminal node the reliability of the associated system component is simply the node output designated as O_i, as shown in figure 2. The neural tree output at the root node follows the trend of the reliabilities; that is, the higher the reliabilities, higher the reliability at the root node. This is more explicitly explained by the following example. Consider that neural tree represents the safety of a vehicle. If all the component reliabilities of the vehicle are high, then accordingly the safety of the vehicle is high at the output and vice versa. At any point in the tree structure, if the reliability of a component goes high, then the reliability at the output bound to be high too provided the component has relevance to the output. The weights w_{ij} represent the discrete possibility distribution of the reliabilities at the node outputs. This is based on evidence theory [9] and interpreted as the degree of belief (based on available evidence); that is, it is based on the knowledge already available. This possibility distribution shape the Gaussians as continuous possibility distribution via the parameters of the discrete possibility distribution, as this is seen in (7), (8) and (9). After scaling we have a Gaussian width σ_j/w_{ij} of the possibility distribution. In this neural tree structure, only the root node performs a simple weighted summation of the inputs coming from the immediate layer below. Terminologically, this is the de-fuzzification process for the final outcome. Therefore the weight connected to the root node sums up to unity. By means of the above described approach, the locations of the Gaussian possibility distributions at the non-terminal nodes are well-defined. Each input to a node has contribution to the output of that node based on logic AND operation. The centre location of *i*-th Gaussian possibility distribution is selected as w_{ij} so that certain features described above are maintained. At this point a few observations below are due.

If a w_{ij} is zero, this means the possibility zero and consequently the associated reliability has zero effect on the node output and thus also the system output. Conversely, if a w_{ij} is unity, this means the possibility is highest among the competitive weights directed to the same node and consequently the associated reliability has the largest importance on the node output via respective AND operation; that is, the value of the associated reliability is very important and a small change about this value can have big impact at the node output. This is the importance mentioned about, above. If a w_{ij} is somewhere between zero and one, this means the associated reliability has some possible effect on the node output determined by the respective AND operation via (7) or (8) whatever applies. In this way, the domain knowledge is integrated into the reliability calculations thereby the insufficiency of the reliability data is compensated for more accurate reliability estimations.

Until now, in the above considerations, the width of the Gaussian possibility distribution given by (9) assumed to be known. However, what is believed to be known is w_{ij} but not σ_j. The determination of σ_j is explained below.

The neural tree connection weights having been determined with the consideration of evidence theory, the reliabilities of the node outputs are estimated for some contrived scenarios. We can consider that the neural tree structure represent an event tree [10] giving assessment about the reliability of the system in the presence of initiating events at the input. The general properties of the present neural tree structure relevant to the width determination are as follows.

- If an input of a node is small (i.e., close to zero), the output of the node is also small complying with AND operation
- If a weight w_{ij} is low (i.e., possibility associated with the component reliability is low), the associated w_i or O_i cannot have significant effect on the node output. This means, quite naturally, such inputs can be ignored.
- If an input of a node is high (i.e.,close to unity), the output of the node is also high complying with AND operation (i.e., if the event initiating component reliability is high, the reliability representing the node output is also high)
- If a weight w_{ij} is high (i.e., possibility associated with the component reliability is high), the associated w_i or O_i can have significant effect on the node output unless w_i or O_i are too low or too high. This is because if w_i or O_i are too low or too high the situation becomes the same as considered above (first and third options, respectively).

With the above observations and considering the properties represented by figure 3, one can conclude that the input and the output data sets given in Table I and Table II, respectively comply with the general properties given above. Since we are considering possibilistic statements, the approximation imposed by these data sets are valid and the probable imprecision is absorbed by the pertinent possibility distributions.

TABLE I
DATASET AT THE NEURAL TREE INPUT FOR CONSISTENCY CONDITION

.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
.3	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3
.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4
.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6
.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7
.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8
.9	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9

TABLE II
DATASET AT THE NEURAL TREE OUTPUT FOR CONSISTENCY CONDITION

.1	.2	.3	.4	.5	.6	.7	.8	.9

By means of the neural-tree a holistic approach is implemented in a complex system for reliability assessment. Because of a number of event initiating occurrences and their probable interdependence, the actual reliability calculations can be formidably difficult, on one hand. On the other hand, to make such assessments again and again for different input composition is another issue of concern. In contrast with this, to establish the tree one time and to carry out the reliability assessment as an outcome of the tree is appealing and exiting. Based on the data set in Tables I and II, the neural tree is trained as a network so that appropriate σ_j values pertinent to the respective nodes are determined. The information embedded in these tables is referred to as *consistency conditions*, in this work. As result of the training, the consistency conditions of the tree are established and all the Gaussian widths are determined. An example neural tree structure is shown in figure 4 where certain input composition with some failures causes some failure possibility at the root node. The situation is reminiscent of fault tree analysis (FTA) in the reliability science. Here at one hand we are dealing with reliabilities via possibility distributions. Therefore, the method of the analysis is quite different than that of the FTA. However in both cases the system reliability is computed and the possibility distribution absorbs the imprecision of the reliability data. It is common that in both cases the reliability assessment is made not only for the system output but also for various locations of the system, as by product. Possibilistic reliability is an essential support for FTA, as this is discussed in section V. The training for consistency conditions defined above can be carried out in different forms to improve the performance of the reliability assessment. In one option, if all the inputs to the tree are uncorrelated and the tree is in the form of several branches, then the training is performed for each branch. A branch is defined as independent set of nodes connected to the root node. The output of a branch is taken as the node one below the root node.

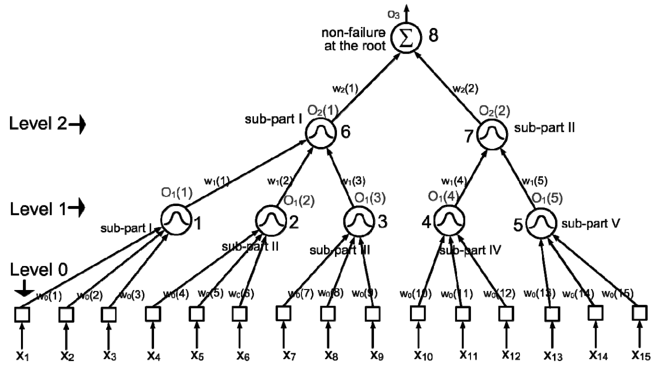

Fig. 4. An example of a neural tree employed for design

For example in figure 4, there are two branches. The branch at the right side has 9 inputs and the output node is designated as node number 6. The other branch has 6 inputs and the output node is designated as node number 7. When the consistency conditions are established for all branches, it includes inherently the root node also. In another option, if the inputs to the tree are correlated, then to allow for this correlation, the training for the consistency condition includes also the root node meaning that for the whole tree structure one common training is performed.

So far a novel neural tree is described in detail for a system reliability modelling based on possibilistic knowledge integration into the system. In neural tree the possibilistic reliability information flow from input to output is according to fuzzy logic AND operations in the form of *rule chaining*, that is, a *consequent* or a rule output is an input for another rule in the form of a chain where the Gaussian radial basis functions play the role of possibility distributions.

IV. COMPUTER EXPERIMENTS

The computer experiments are carried out according to the fuzzy neural tree shown in figure 5. The weights of the tree structure are given in Table III together with the input composition indicated at level 0.

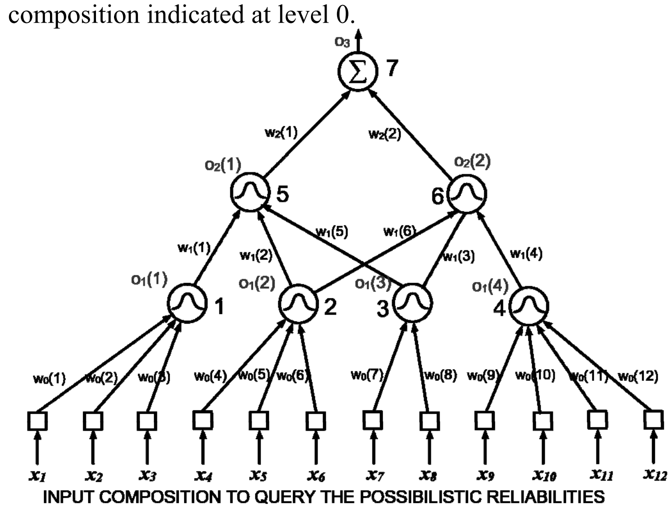

Fig. 5. The neural tree used in computer experiments for possibilistic reliability assessment

Considering that the neural tree belongs to neural networks paradigm, it can be trained using one of the neural network training algorithms. Taking the data set given in Table I as input and the data set given in Table II as output matching the input, a training is implemented; namely there are 9 data samples of size 12 at the input and 9 data samples of size 1 at the output subjected to back-propagation. The adaptive learning results from the data sets are given in Table IV.

TABLE III
WEIGHTS OF THE NEURAL TREE FOR THE DESIGN PERFORMANCE

Level 2	.40	.60										
Level 1	.60	.40	.45	.55	.50	.80						
Level 0	.60	.40	.30	.60	.40	.30	.60	.10	.20	.25	.35	.20

TABLE IV
ADAPTIVE LEARNING RESULTS FROM TABLES I AND II

Given for all inputs and output	Approximation	Error
1.000000E-01	1.657328E-01	-6.573278E-02
2.000000E-01	2.093839E-01	-9.383857E-03
3.000000E-01	2.765190E-01	2.348101E-02
4.000000E-01	3.726155E-01	2.738446E-02
5.000000E-01	4.931522E-01	6.847829E-03
6.000000E-01	6.160555E-01	-1.605546E-02
7.000000E-01	7.103201E-01	-1.032013E-02
8.000000E-01	7.771928E-01	2.280718E-02
9.000000E-01	9.249367E-01	-2.493668E-02

The input composition querying the neural tree at the non-terminal nodes is given in Table V and the resulting reliabilities at the root node are shown in Table VI

TABLE V
THE INPUT COMPOSITION QUERYING NEURAL TREE

Level 0	1.0	.90	1.0	1.0	.90	1.0	.95	.90	.85	.95	.90	.85

TABLE VI
POSSIBILISTIC RELIABILITY AT THE NODES

Node 7 (root node)	..990			
Nodes 5 and 6	..978	1.		
Nodes 1-4	.837	.992	.995	.984

From Table VI, we observe that node number 1 has relatively low reliability and if the overall reliability is aimed for improvement, then the possibility distribution of the relevant weights of this node should be considered in the first place. These results are important indicators for the reliability improvement of the system, in particular node 1, in this case. It is worth to point out that, there are three inputs to this node. Two inputs, i.e. first and third are unity and therefore each delivers 1 at the output so that at the AND operation only the second input can cause change at the node output. The second input has reliability as 0.9 and the associated possibility is 0.4. With these values, the Gaussian possibility distribution has a width given by (9) as $\sigma_I/0.4$ where σ_I is adaptively determined by training. This is illustrated in figure 6.

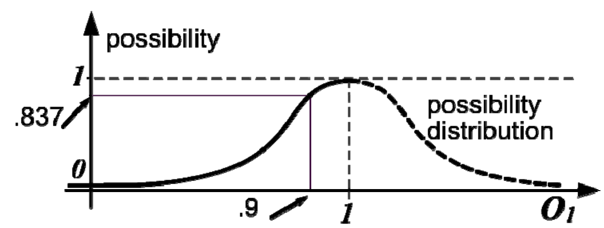

Fig. 6. Analysis of the result for node 1

From the figure, the possibilistic reliability is computed as 0.837 while the node input reliability is 0.9. The other inputs deliver 1 so that the AND operation results in 1×1×.837=.837, as indicated in Table VI.

From Table VI, it is possible to monitor at which locations the reliabilities are relatively low. For instance at the node 5, the reliability is given as .978 whereas at node 6, it is 1. It is to emphasize that once the neural tree is established with accurate reliability centred parameterisations. The reliability assessment for the various parts of the system with any input reliability composition becomes an easy task, which is formidable otherwise. Also, by sensitivity analysis, the reliability sensitivity of the output with respect to the input can be determined. This is an additional reliability data about the system with respect to its inputs. Such inputs can be thought of some event initiating occurrences stemming from the other system components which are not included in the tree structure.

V. DISCUSSION

Possibilistic reliability assessment in engineering systems is an important step for reliability improvement in such systems. Especially in complex man-machine interfaced systems, the reliability assessments for various parts of the system are also a complex issue. The complexity is twofold. On one hand, the system is complex so that after decomposition, the reliability assessment is still a complex task. On the other hand, especially in such systems failure data are very limited because the occurrence of failure events is limited and even does not exist. The reliability calculations require desperately actual data to provide useful realistic outcomes. The probability distributions for describing failures used can be seen in two categories as discrete distributions and continuous distributions. For discrete distribution Binomial and Poisson distributions are common. For continuous distributions which are essential for reliability assessment of a continuous working system, the Erlangian and exponential distributions are of primary concern. The Erlangian distribution is the time-dependent form of the Poisson discrete distribution. The Erlangian distribution is used for failures for which the hazard rate $\lambda(t)$ is constant, i,e., λ. To derive the distribution, we consider a Poisson distribution of the form

$$P(r) = \frac{e^{-\mu} \mu^r}{r!} \tag{14}$$

where r is exact number of events (failure occurrences, in the present context) ; μ, is the most probable number of occurrences of the event.; $P(r)$ is the probability associated with r. We recognize that the mean number of failures μ is the product of λ and time. The probability of exactly r failures occurring in time t is then given by

$$P(r) = \frac{e^{-\lambda t} \lambda t^r}{r!} \tag{15}$$

From (15) we can obtain the failure probability density f(t) for the r-th failure in dt about t as follows. For the system to have undergone (r-1) prior failures so that it is ready to fail for the *r-th* time with a conditional probability λ Thus the Erlangian distribution follows from (15) as

$$f(t) = \lambda\, P(r-1, t) = \frac{\lambda\, (\lambda t)^{(r-1)}\, e^{-\lambda t}}{(r-1)!}, \quad \lambda > 0, \tag{16}$$

The Erlangian distribution is valid for an integer number of failures r. The most important special case is for $r=1$, in which case the exponential distribution is obtained:

$$f(t) = \lambda\, e^{-\lambda t} \tag{17}$$

which is used for analyzing the purely random failure of a system characterized by a constant hazard rate.

With the above considerations, the possibilistic reliability assessments can be of marked value in two ways. Firstly, with such simplifications based on Erlangian failure assumptions, the reliability assessment of complex systems becomes formidable task in the presence of the scarcity of the failure data. Therefore the possibilistic reliability assessments are important support for reliability assessments, indicating the places where (for which components) reliabilities are relatively low. Secondly, in a complex system one is mainly interested in the reliability of critical components rather than the whole system. This is necessary especially for maintenance purposes where the sustained working condition of the system is desired for economic reasons if the system is not critical in which case human lives are involved. For effective maintenance purposes one should identify at first the critical component or components by means of reliability considerations which can be formidable as explained above. In this case, possibilistic reliability considerations can be of great help indicating the locations at once where the non-failure possibilities are relatively low. Here the fuzzy neural tree is an effective tool for this assessment since it does not imperatively require reliability data but expert judgement to form the tree structure. Definitely, if reliability data is available, then some reliability computations and considerations can be of significant support to form the tree. Such support comes from the FMEA analysis, as this is mentioned before.

VI. Conclusions

A fuzzy neural network as a possibilistic reliability network is described where the definition of possibilistic reliability in this context is clearly defined. The novelty stems from the particular neural tree structure bearing fuzzy logic concepts and the associated details which make altogether the possibilistic reliability assessment work. This is demonstrated by means of computer experiment and integrating the novel "possibilistic reliability" concept into a novel fuzzy neural network introduced for other soft computing application [1] which deals with design. The motivation of the possibilistic reliability research presented in this work is especially for the assessment of the building technological systems' services reliability viewpoint. At the same time, it presents a demonstration of soft computing in building construction especially in the context of maintenance. The present work is a novel significant step along this direction.

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