
Visual perception model for architectural design

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Abstract: A model of human visual perception is presented. It is based on probability theoretic considerations. A probabilistic approach is suitable to model perception, as it can absorb the bio-induced complexity of the human vision process. In this way the perception and related concepts are quantified. The results provided by the model corroborate with common human vision experience confirming the validity of the model. The significance of distance on perception is described and exemplified in two applications for architectural design.

Keywords: early vision; early vision probability; perception of ceiling; perception measurement; perception modelling; scene description by perception; visual attention; visual perception.

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1 Introduction

Human visual perception is an important issue in architecture. This is because spatial experience and cognition are mainly based on information obtained via visual perception and they form a significant aspect of architecture. Developing a model of visual perception is a relevant issue for architectural design, as it may help to increase awareness for perceptual implications of design decisions. Modelling human perception is challenging mainly because it involves not only the eye, but also the brain. The final ‘seeing’ event occurs in the brain. Brain processes are responsible for a common phenomenon we encounter practically every moment, although it may remain unnoticed: we overlook items in our environment, although they are visible to us. This vision experience can be easily verified. When we view a scene for some time, and then try to remember the items of which the scene consists we may not remember all of them. Although light that reflected on the objects located within the visual scope reached the retina of the observer, he/she may not be sufficiently aware of all of the objects to remember all of them. This *uncertainty* is a characteristic property of human perception. It is presumably due to the way the human memory is built up and operates. In this respect the human vision system clearly differs from an optical system like a camera system. When we say that we perceived something, the meaning is that we are able to recall relevant properties of the item. What we cannot remember, we cannot claim we ‘saw’, although we may suspect that corresponding image information was on our retina. Concerning the effective result such an event of being unable to remember is indistinguishable from the event that we did ‘not see’. The commonly experienced uncertainty of ‘seeing’ has never been explained precisely. This is mainly due to the complexity of the brain processes involved in human vision, which are not sufficiently known, yet. The geometric properties of the environment fundamentally influence perception. We can easily verify this considering that we are usually aware of an object when it is located nearby our eyes; however we may easily ‘overlook’ the same object if it is placed at great distance. The influence of geometry plays a particular role during the phase known as *early vision* in the literature (Adelson and Bergen, 1991; Bertero et al., 1988; Pappathomas et al., 1995; Van Hemmen et al., 2001). This is the process in which an observer builds up an initial comprehension of his/her surroundings. The initial awareness of the environment is decisive for later stages in vision and ensuing cognition. However, the influence of geometry in early vision has not been precisely known until now. The main goal of the present paper is to gain insight into the role geometry plays in early vision, so that design decisions can be taken with increased awareness and cognition related knowledge can be applied based on a solid foundation.

In this paper, a novel model of early human vision is described. The model quantifies the degree an observer becomes aware of environmental objects during early vision. In this way the commonly experienced *overlooking* of visible information in perception is explained. The model is based on probability theoretical considerations, so that it can duly deal with the complexity of the human vision process. To our best knowledge, this is a novel probabilistic approach to perception. Please note that the model is not *statistical* as this is conventionally the case in literature with Bayesian inference (Knill and Richards, 1996). Before developing the model, due to the diversity of existing approaches related to vision and visual perception a reasonably comprehensive introduction is provided to be explicit with respect to the contribution of the present research.

Human visual perception has been considered as the reconstruction of a three-dimensional scene from two-dimensional image information (Bigun, 2006; Marr, 1982; Poggio et al., 1985). This *image processing* approach attempts to mimic the biological and neurological processes involved in vision, with the retinal image acquisition as the starting event. This is done by taking a two-dimensional image as a starting point in the model. However, modelling by means of algorithmic counterparts the sequence of eye/brain processes that treat the retinal image is a formidable endeavour. This is because these processes are complex and conditional, while the brain processes play a decisive role. In modelling human vision, the involved brain process components as well as their interactions should be known with certainty if a deterministic approach is followed. The existing knowledge on this matter is currently not sufficient, so that deterministic approaches do not explain the characteristic uncertainty of perception, which is commonly observed. Well-known observations of visual phenomena, such as *depth from stereo disparity* (Julesz, 1964), *Gelb effect* (Cataliotti and Gilchrist, 1995), *Mach bands* (Ghosh et al., 2006), *gestalt principles* (Desolneux et al., 2003), *depth from defocus* (Favaro and Soatto, 2005; Pentland, 1987), *depth from focus* (Subbarao and Choi, 1995) etc., reveal components of the vision process that may be algorithmically mimicked. However, it is unclear how they interact in human vision. In particular it remains mysterious how the combination of the modelled components results in the uncertainty of perception.

In his *ecological* or *direct* approach to perception Gibson (1986) identifies the shortcomings of the image as a starting point of a perception model. He proposes to consider perception as *direct pickup* of environmental information by means of successive eye fixations. The concept is appealing, however, neither Gibson, nor his followers (Turvey and Shaw, 1999) provide precise insight about the nature of fixations and how to model the pickup; as result of this perception remains a verbally defined phenomenon. Ecological optics does not explain precisely how the commonly experienced overlooking of visible information occurs. However, the authors of the present work agree with Gibson's view, which reads "in contrast to its use in communication theory information should be construed as specificity of the useful rather than as the uncertainty of the specific. This sense of information is the best way to understand how perception could control behavior" (Turvey and Shaw, 1999). In the present work, early vision information is construed as specificity, which is the *awareness* of the environment. This awareness extends even to infinity in space as scope of vision. The imprecision involved in the cognition of this awareness is subjected to probabilistic modelling, as this will be described in the next section.

Inspired by Gibson's work, in the context of event perception research a concept related to human vision has been introduced as *isovist* (Benedikt and Burnham, 1985). An isovist is the array of the distances that span between an observer's viewpoint and the surfaces of the environment that are visible from the viewpoint. Isovists are used for spatial analysis in urban, architectural and landscape design, as well as in psychology research (Batty, 2001; Franz et al., 2005). An isovist can be considered a model of the paths light directly travels from the environmental objects to the retina of an observer. It is not a model of visual perception. Perception involves the processing of visual stimulus yielding the mental realisation of environmental objects and events with its characteristic uncertainty. Works based on isovist omit this essential aspect of perception. In the domain of geographical information science a method termed *visibility graph* has been introduced

(De Floriani et al., 1994). This is a graph containing as nodes a number of locations that are mutually visible in an environment. Visibility graphs are used in urban and architectural design for spatial analysis (Turner et al., 2001), where they are also used to approximate the volume defined by an isovist. A visibility graph can be seen as a model of the space spanning between an observation point and the visible surface forming the environment. Clearly, it is not a model of visual perception. It does not model the vision process with its characteristic uncertainty.

In the domain of virtual reality, perception models have been developed. In Herrero et al. (2005) the *visual acuity* of the human eye as well as the *minimum eye resolution* are simulated to model visual perception. Positions in the environment, which are located within a limited central region of the visual scope, are considered to be ‘perceived’, while objects located outside this region are ‘not perceived’. The approach ignores the common phenomenon that we overlook items in our visual scope, even when they are located within a limited central aperture in our visual scope, although such overlooking may be unlikely to occur.

Brain researchers trace visual signals as they are processed in the brain. A number of achievements are reported in the literature (Cowey and Rolls, 1974; Hecht-Nielsen, 2006; Hubel, 1982; Taylor, 2005; Wiesel, 1982). However, due to complexity there is no consensus about the exact role of brain regions, sub-regions and individual nerve cells in early vision, and how they should be modelled. The neuroscience models are all different. This is because there is enough modality of the brain to accommodate all conclusions directed by the research. Therefore, they are not unified in understanding a particular brain process like *perception* on a common ground. The difficulty in modelling perception deterministically is increased by the involvement of attention mechanisms in perception. Attention mechanisms are driven by diverse conditional interests of the human being (Treisman, 2006; Treisman and Gelade, 1980). Experiments showed that what is remembered from a scene depends on attention (O’Regan et al., 2000; Rensink et al., 1997). However, despite experimental research on attention and perception it remains uncertain what the concepts exactly are, and how they can be modelled quantitatively. The uncertainty originates from the difficulty to distinguish precisely among the various factors involved in the viewing experience that range from the geometry of the environment to personal preferences. Identification and pinpointing of the influence of attention on the visual processing in the brain is formidable due to the intimate relation between attention and vision. An observer can exercise his/her bias or preference for certain information within the visual scope only when he/she already has a perception about the scene, with respect to where potentially relevant items exist in the visible environment. This information is not available in early vision by definition. The early vision phase is omitted in the works on attention mentioned above. Without precise understanding of the early stage of vision, identification of attention in perception, that is due to a task specific bias, is limited. This means, without a model of the initial stage of perception its influence on later stages is uncertain, so that the later stages are not uniquely or precisely modelled and the attention concept is ill-defined. Since attention is ill-defined, ensuing perception is naturally also merely ill-defined. The relation between environmental stimulus and the mental event of perception remains unknown.

As a recapitulation of the previous section please note that visual perception and related concepts have not been exactly defined but qualitatively considered among the general vision related issues. Therefore, the perception phenomenon is not explained

precisely or quantified. Consequently the use of the perception concept in architectural design is limited to application of personal experience or knowledge obtained from experimental data.

In the present paper, a newly developed model of perception during early vision is introduced. In this model visual perception is put on a mathematical foundation. This is accomplished by means of the probability theory. The work concentrates on the early stage of the human vision process, where an observer builds up an unbiased initial understanding of the environment, without involvement of task-specific bias. In this sense, it is an underlying fundamental work, which may serve as basis for modelling later stages of perception, which may involve task specific bias. The probabilistic model can be seen as a unifying model as it unifies synergistic visual processes of humans, including physiological and neurological ones, as well as philosophical aspects of vision. Interestingly this is achieved without recourse to neuroscience or biology. In the present approach the components involved in human vision are modelled as a whole system instead of modelling each component individually. Thereby the model bridges from the environmental stimulus to its mental realisation during early vision. The model has ample space to accommodate neuroscience, biology or other interdisciplinary aspects of perception without any restriction. At this point the definitions of attention and perception during early vision are not elaborated further, since this will be established naturally as a result of the vision model, which is described in the next section.

The novel model gives two advantages:

- the perception and related phenomena in early vision are understood in greater detail, and some common reflections about them are substantiated
- the model can be effectively introduced into advanced implementations, such as architectural design, since perception is quantified via a probability.

From the introduction above, it should be emphasised that the research presented here is geared to demystify the concepts of perception and attention during early vision from their verbal description to a scientific formulation. Due to the complexity of the issue, such formulation apparently is not reported to date. This is accomplished by not dealing explicitly with the complexities of brain processes or neuroscience theories, about which more is unknown than known, but by incorporating them into perception via probability. The vision model is derived based on common human vision experience explaining the causal relationship between vision and perception at the very beginning of the human vision process. For this very reason, the presented vision model modestly precedes commonly existing works in the sense that, they can eventually be coupled to the output of the present model. The novel model introduced here is based on a premise, which is presented in the following section, and it explains a number of perception related phenomena that we commonly experience as result of our vision process. In this way, the integrity of the results from the model is conformed by means of the practical implications.

2 Probabilistic perception model

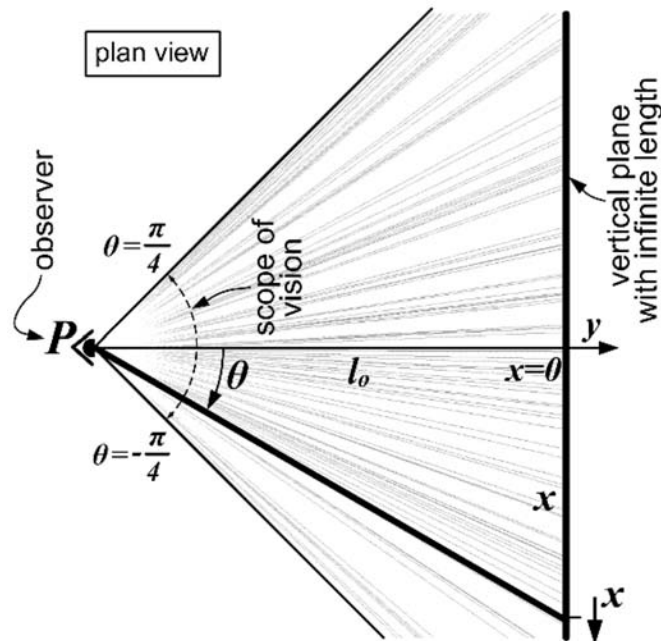
Vision is our essential source of information while we interact with our environment. Based on vision, many derivative concepts of vision can be defined, such as visual

perception, visual attention, visual privacy, and so on. However, since these concepts, which have been derived from the vision process, have only been verbally described until now, they are not precisely defined in the literature, although in essence there is a rough consensus. Having noticed that, this work endeavours to establish a mathematical model of early vision, so that the ensuing concepts of vision derivatives are mathematically defined. As a result of this it is foreseen that several elusive concepts, like visual perception and visual attention, will no longer be elusive but subject to quantification and computation. By doing so, several vision-related concepts can be effectively introduced into advanced implementations that involve early vision, like scene description by perception or visual openness measurement for design, having insight into the role of perception in such tasks.

Early vision involves image acquisition with the eye and interpretations in different regions in the brain. Since this process is formidably complex and its details are not known, its modelling is formidable when deterministic methods are used. The well-established probability theory is particularly suitable to handle such high complexity and uncertainty. The main principle of modelling by probability theoretic considerations is to absorb complexity and uncertainty into probability. With respect to early vision such a model is appealing, as it can absorb the complexity of the eye/brain process by the probability theoretic considerations, resulting in a match between model outcome and the perception phenomena we commonly experience.

We start with the basics of the perception process during early vision using a simple, yet a fundamental geometry. This is shown in Figure 1.

Figure 1 Plan view of the basic geometric situation of perception during early vision; P represents the position of an observer, who is viewing a vertical plane from a distance l_o ; the scope of vision is also indicated, as well as a number of randomly generated vision rays (grey lines)



In the figure, an observer is facing and viewing a vertical plane from the point denoted by P . By viewing, the observer has the chance to receive visual data from all directions within his/her scope with equal probability in the first instance. That is, the observer pays attention to all locations on the plane with uniform probability density with respect to the angle θ that defines the viewing direction, without any preference for one direction over another. This premise ensures that there is no visual bias at the beginning of perception with respect to the direction, from which environmental information may be obtained. Therefore the model concerns the very starting instance of a vision process, before any directional preference for certain information in the environment can be exercised, such as in visual search or object recognition for example. Before an observer has information about the environment he/she is about to experience visually, there is no base to justify any preferred direction in the view, in which attention should be paid. If we were able to specify in advance a certain direction in which we expect to have more environmental information, this bias must be based on previous information about the scene. However, as the environment is unknown to the observer in early vision, such information is not available and hence visual attention is paid equally for any direction within the scope. This is the premise of the model, and it is based on basic human vision experience. It is not an ad hoc approach trying to explain a certain vision phenomenon. Instead this is a starting point from scratch to arrive at some results through further derivations, which follow. Therefore the justification will follow afterwards.

The premise entails that the probability of obtaining environmental information is the same for each single differential visual resolution angle $d\theta$. This means that the probability density function (pdf), which belongs to the angle θ is uniformly distributed. This is given by Equation (1). Assuming the scope of vision is defined by the angle $\theta = \pi/4$, the pdf f_θ is given by:

$$f_\theta = \frac{1}{\pi/2} \tag{1}$$

This is shown in Figure 1, where the scope of vision is taken as $-\pi/4 \leq \theta \leq \pi/4$. To illustrate the implication of the premise a number of ‘sight-lines’ are shown in the figure that are generated based on the uniform probability density given by Equation (1). Each of them represents an event of paying attention to a certain location on the plane. The angle θ is a *random variable* in the terminology of the probability theory. Since θ is trigonometrically related with each point on the plane, the distance x , which is indicated in Figure 1, is also a random variable. The pdf $f_x(x)$ of the random variable x is computed as follows.

$$tg(\theta) = \frac{x}{l_o} \tag{2}$$

Applying the theorem on the *function of random variable* (Papoulis, 1965): to find $f_x(x)$ for a given x we solve the equation:

$$x = g(\theta) = l_o tg(\theta) \tag{3}$$

for θ in terms of x . If $\theta_1, \theta_2, \dots, \theta_n$, are all its *real* roots,

$$x = g(\theta_1) = g(\theta_2) = \dots = g(\theta_n) = \dots$$

Then

$$f_x(x) = \frac{f_\theta(\theta_1)}{|g'(\theta_1)|} + \frac{f_\theta(\theta_2)}{|g'(\theta_2)|} + \dots + \frac{f_\theta(\theta_n)}{|g'(\theta_n)|} \quad (4)$$

Clearly, the numbers $\theta_1, \theta_2, \dots, \theta_n$, depend on x . If, for a certain x , the equation $x = g(\theta)$ has no real roots, then $f_x(x) = 0$. According to the theorem above we write:

$$g'(\theta) = \frac{dx}{d\theta} = \frac{l_o}{\cos^2(\theta)} = l_o [1 + \tan^2(\theta)] = l_o \left(1 + \frac{x^2}{l_o^2}\right) \quad (5)$$

$$\theta_1 = \arctg(x/l_o) \quad (6)$$

$$f_x(x) = \frac{f_\theta(\theta_1)}{|g'(\theta_1)|} = \frac{l_o}{\pi(l_o^2 + x^2)} \quad (7)$$

for the interval $-\infty \leq x \leq \infty$. (Ciftcioglu et al., 2006). For this interval, the integration below becomes:

$$\int_{-\infty}^{+\infty} f_x(x) dx = \frac{l_o}{\pi} \int_{-\infty}^{+\infty} \frac{dx}{x^2 + l_o^2} = 1 \quad (8)$$

as it should be as a probability density function. For the interval

$$-l_o \leq x \leq l_o \quad (9)$$

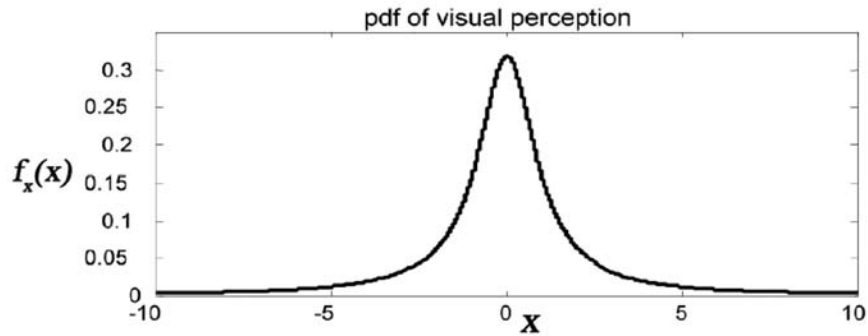
$$f_x(x) = \frac{2}{\pi} \frac{l_o}{(l_o^2 + x^2)} \quad (10)$$

so that

$$\int_{-l_o}^{+l_o} f_x(x) dx = \frac{2l_o}{\pi} \int_{-l_o}^{+l_o} \frac{dx}{x^2 + l_o^2} = 1 \quad (11)$$

as one should expect. The plot of $f_x(x)$ as a function of x is shown in Figure 2.

Figure 2 Probability density function of perception during early vision along the x axis for $l_o = 2$



Based on the results of the basic theoretical considerations above, some vision related concepts can be terminologically defined in mathematical terms. Namely, early vision is the ability to see. All vision-related concepts, such as attention, perception, visual openness and others are the sub-areas of vision. This means, vision is the essential necessity for the further considerations in these sub-areas. The probability density function $f_x(x)$ derived in the preceding subsection corresponds to *visual attention*, which is a well-known concept in cognitive psychology (Itti et al., 1998; Posner and Petersen, 1990; Treisman, 2006; Treisman and Gelade, 1980), yet never formally defined with consensus. From the derivations above it should be noted that visual attention is given by a probability density within an infinitesimally small spatial region, and per unit interval. The integration of the probability density function $f_x(x)$ within a finite length interval yields a probability. This probability quantifies perception, which is eventually referred to as *early vision probability*. The probability is associated to the event that an observer obtains information from a region in the environment. This means perception is modelled as a probabilistic event. The definition of attention and perception given above may not exactly coincide with those used in the psychological literature due to generally vague and verbal definitions in the latter case. However, the mathematical formulations presented above are decisive, and they are subject to adoption into diverse fields. The integration of visual attention in a finite small interval where $f_x(x)$ is approximately constant, gives early vision probability (P)

$$P = \int_{\Delta x} f_x(x) dx \cong f_x(x) \Delta x \quad (12)$$

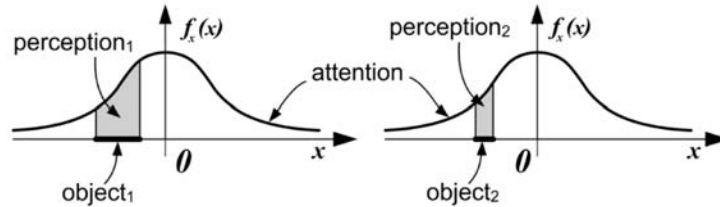
so that the probability becomes proportional to attention in this case. For the forward direction of viewing the infinite plane where $\theta \approx 0$ the vision probability becomes

$$P \approx \frac{2}{\pi} \Delta\theta \quad (13)$$

where $\Delta\theta$ is the vision resolution of the human eye. Although in the theoretical development the region over which attention is integrated to yield the early vision probability can be infinitesimally small, in the practical case the region is restricted by the human eye resolution, which is defined by the differential angle $\Delta\theta$.

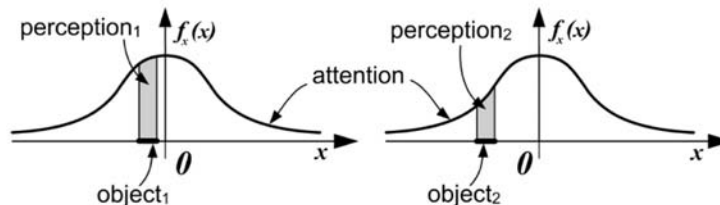
The results from the above computations are in conformity with the common human experience in early visual perception. Namely, for the plane geometry shown in Figure 1 the visual attention and thereby vision probability is strongest along the axis of the cone of vision, relative to the side directions. This means that details of the plane that are directly in front of the observer have a greater chance to be seen, and hence remembered, than the side parts. One can easily verify the results above from the probabilistic vision model intuitively by basic vision experience. For instance, in a scene, when one considers only the geometry and not any other aspects like illumination or colour, comparatively an object with bigger dimensions is more likely to be seen, and therefore early vision probability of this object is high. If there is more than one object in the scene, then the final seeing action for these objects will be merely subject to probability, which means subject to perception. This is because the attention during early vision, that is the discrete probability density, would be distributed among these objects. The bigger object would be more likely to be seen in this case. This is shown in Figure 3.

Figure 3 Sketch showing the difference in early vision probability of two objects with different size positioned approximately at the same location along the x axis given in Figure 1



In case we position an object at different places along the x axis given in Figure 1, the closer the object is to $x = 0$, the greater is its early vision probability. This is shown in Figure 4. As the vision probability is obtained through integration of attention over a particular spatial domain, the perception model is closely related to gestalt principles. Gestalt theory addresses the phenomenon that an observer experiences a number of visual items as belonging together, so that they form another visual entity, namely a group. The group is referred to as *gestalt*. With respect to the role of gestalt theory for visual perception it should be noted that an observer groups visual items based on perception as introduced here. This means an observer should have mentally realised the elemental visual items that form a group beforehand. The number of domains and their shapes in a scene to be integrated is subject to probabilistic considerations. In these considerations, the elementary probability of a gestalt quality denoted by p can be assessed by means of the perception considerations presented here, and further computations can be carried out with underlying probability theory (Desolneux et al., 2003). In this way seamless coupling of the gestalt theory to the perception model presented here is an interesting relevance.

Figure 4 Sketch showing the difference in early vision probability of two objects with the same size positioned at different locations along the s axis given in Figure 1



3 Application of the perception model in architectural design

3.1 Scene description by perception

The probabilistic perception model for early vision can be used to compare architectural spaces with respect to perception defined above in mathematical terms. For this purpose the dependency of the vision probability on distance is demonstrated below. Let us consider the probability density $f_x(x)$ that is the early visual attention in the case of viewing an infinite plane remains constant. This constant attention is denoted by c . Its unit is probability per unit length. Taking the visual scope as $-\pi/2 \leq \theta \leq +\pi/2$, from Equation (7) we write (Ciftcioglu et al., 2006):

$$\frac{1}{\pi} \frac{l_o}{x^2 + l_o^2} = c \tag{14}$$

which yields

$$x^2 + \left(l_o - \frac{1}{2\pi c}\right)^2 = \frac{1}{(2\pi c)^2} \tag{15}$$

Taking both x and l_o as variables Equation (15) represents a circle in a coordinate system formed by x and l_o axis, with the origin at the position of the observer. This is illustrated in Figure 5, where the position of the observer is denoted by P . The corresponding early attention probability density is also shown in the figure, where the attention threshold $f_x(x) = c$ is indicated. This circle is termed as *circle of attention*. Along the circle circumference the attention has a constant value; namely c . The inner part of the circle corresponds to the space, where the perception during early vision is deemed to be significant and relatively insignificant outside of the circle. Outside of the circle the attention is $f_x(x) < c$. In this sense c is a threshold level for attention, below which perception is considered not to occur. The attention threshold concept is necessary to apply for consistent comparison of scenes. Let us compare the vision probability of a scene viewed from different distances. For comparison, the constant attention threshold c is taken to be the same for both cases. This yields the same circle of attention in both cases. This is shown in Figure 6, where the scenes in question are represented as bold vertical lines within the circle of attention seen in the figure.

Figure 5 Illustration showing *circle of attention*, which represents the probability density function of early visual perception, while the probability per unit length is taken as constant value c

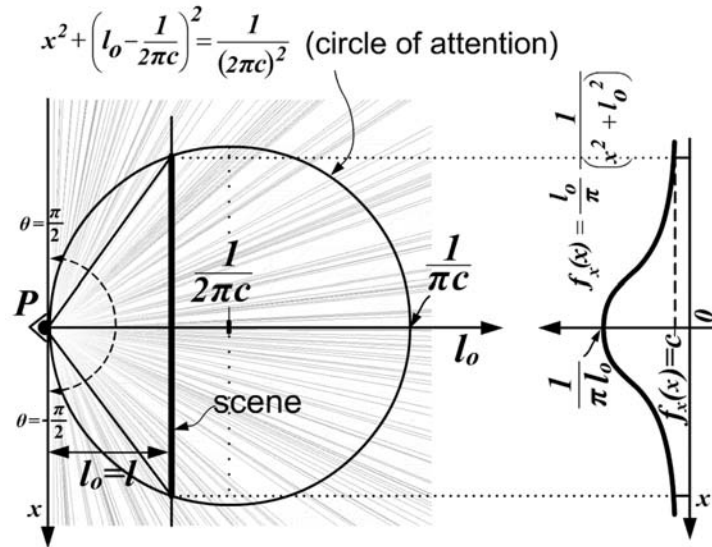
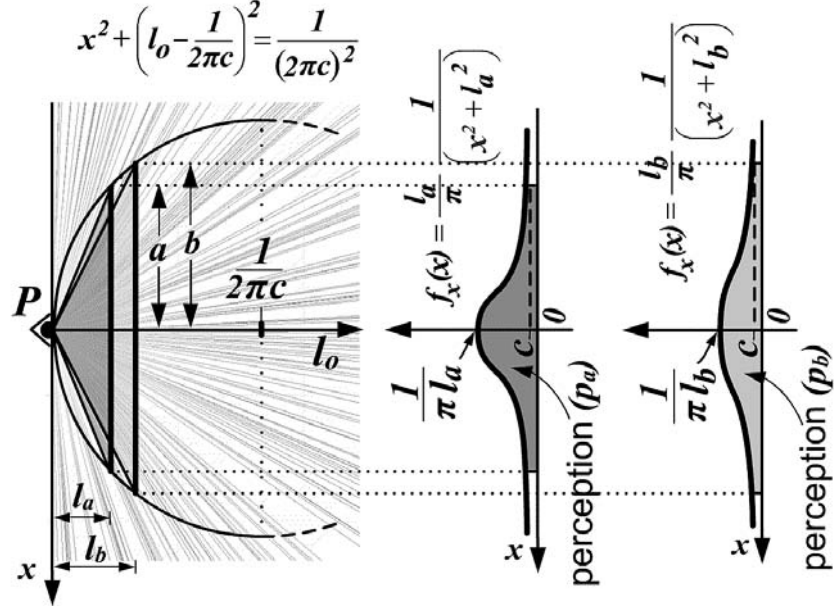


Figure 6 Illustration of the effect of changing the viewing distance from l_a to l_b with respect to the viewed scene



For the sake of clarity of the explanation, first the scene in question is considered as a one-dimensional entity, extending in the x -direction only. The distances between the viewpoint and the scenes are denoted as l_a and l_b . Clearly, $l_a < l_b$. The circle of attention intersects the scenes at different locations, so that the portions of the scene within the circle have different size. The portions are denoted as a and b in Figure 6. The early vision probabilities of the scenes differ depending on the distance the scene is viewed from. In Figure 6 the respective probability density functions $f_x(x)$ that belong to the different distances are sketched. The vision probabilities p_a and p_b respectively in the cases of $l_o = l_a$ and $l_o = l_b$ are indicated by the shaded areas in the $f_x(x)$ vs x plots. The probabilities correspond to the viewed scenes, which are shown within the circle of attention. To obtain the difference of the scenes with respect to their respective vision probabilities p_a and p_b , the latter are computed as follows. For $l_o = l_a$, from Figure 6, the integral of attention, as perception, yields:

$$p_a = \int_{-a}^{+a} f_x(x) dx = \frac{l_a}{\pi} \int_{-a}^{+a} \frac{dx}{x^2 + l_a^2} = \frac{2}{\pi} \operatorname{arctg} \frac{a}{l_a} \quad (16)$$

For $l_o = l_b$, from Figure 6, the integral of attention yields:

$$p_b = \int_{-b}^{+b} f_x(x) dx = \frac{l_b}{\pi} \int_{-b}^{+b} \frac{dx}{x^2 + l_b^2} = \frac{2}{\pi} \operatorname{arctg} \frac{b}{l_b} \quad (17)$$

As an illustrative example, assuming that $a = 0.65R$ where R is the radius of the circle of attention, l_a is computed from Figure 6 as:

$$l_a = R - \sqrt{R^2 - a^2} \cong 0.24R \quad (18)$$

so that from Equation (16) the perception p_a becomes

$$p_a \cong \frac{2}{\pi} \arctg \frac{0.65R}{0.24R} \cong \frac{2}{\pi} 1.22 \cong 0.78 \quad (19)$$

Assuming that $b = 0.8R$, l_b is computed from Figure 6 as:

$$l_b = R - \sqrt{R^2 - b^2} = 0.4R \quad (20)$$

so that from Equation (17) the perception p_b becomes

$$p_b = \frac{2}{\pi} \arctg \frac{0.8R}{0.4R} \cong \frac{2}{\pi} 1.11 \cong 0.70 \quad (21)$$

Summarising the computations, for the geometry in Figure 6, the vision probability is higher for near distance l_a relative to the case with far distance l_b . In the figure the geometrical proportions are given by:

$$\frac{a}{l_a} \cong 2.71 \text{ and } \frac{b}{l_b} = 2.00 \quad (22)$$

$$\frac{l_a}{l_b} \cong \frac{0.24R}{0.40R} \cong 0.60 \quad (23)$$

and the respective perceptions are given by

$$p_a \cong 0.78 \text{ and } p_b \cong 0.70 \quad (24)$$

The result in Equation (24) shows that the perception p_a is more than p_b .

The difference in perception is accentuated by the fact that the scene viewed is more accurately modelled as a two-dimensional surface, in contrast to the computation above, where the scene is modelled as one-dimensional. This is accomplished as follows. In the two-dimensional case, perception is considered in the horizontal and vertical directions. For the horizontal direction, the probability density along the x axis is considered as it is shown in Figure 1. Taking the visual scope as $-\pi/2 \leq \theta \leq \pi/2$ the corresponding attention probability density is (Ciftcioglu et al., 2006)

$$f_x(x) = \frac{1}{\pi} \frac{l_o}{x^2 + l_o^2} \quad (25)$$

and an elemental perception during early vision is

$$P_x = \int_{\Delta x} f_x(x) dx \cong f_x(x) \Delta x \quad (26)$$

For the vertical direction the same geometry shown in Figure 1 applies, but instead of the x -axis, perception is considered along a new axis. The new axis is directed perpendicular

to the surface of the paper, has the same origin as the x -axis, and is termed as z -axis. The perception along the z -direction is taken with the same visual scope as in x direction, so that the corresponding attention probability is:

$$f_z(z) = \frac{1}{\pi} \frac{l_o}{z^2 + l_o^2} \quad (27)$$

and an elemental perception is

$$P_z = \int_{\Delta z} f_z(z) dz \cong f_z(z) \Delta z \quad (28)$$

It is important to note that in the two-dimensional perception case the events of perceiving an object in the z -direction vs x -direction are *independent events*. Therefore, the joint probability of the perception is computed by means of multiplication of the vision probabilities belonging to each dimension given by Equations (26) and (28). Explicitly we write:

$$P_{x,z}(x, z) = P_x P_z = \int_{\Delta x} \frac{1}{\pi} \frac{l_o}{x^2 + l_o^2} dx \int_{\Delta z} \frac{1}{\pi} \frac{l_o}{z^2 + l_o^2} dz \quad (29)$$

The effect of distance on early perception is exemplified in two scenes given in Figure 7 and Figure 8. Each scene corresponds to the same circle of attention shown in Figure 6. In Figures 7 and 8 the entrance hall of a building is shown. In the figures a group of persons and several objects can be seen that form a scene, corresponding to the scenes shown in Figure 6. In Figure 7 the scene is viewed from a near distance corresponding to l_a in Figure 6. In Figure 8 the same scene is viewed from a far distance corresponding to l_b in Figure 6. Please note that in Equation (29) dx and dz are infinitesimally small distance intervals around x and z . In the following perception comparison the joint probability given by Equation (29) is approximated by:

$$P_{x,z}(x, z) \cong \frac{1}{\pi^2} \frac{l_o^2}{(x^2 + l_o^2)(z^2 + l_o^2)} \Delta x \Delta z \quad (30)$$

taking Δx and Δz as sufficiently small distances. In Figures 7 and 8 the degrees of elemental early vision probabilities are given forming an array of numbers. The probabilities belong to patches with the size $\Delta x \Delta z$. A degree is plotted at the centre of the respective patch. The summation of the elemental perceptions yields the resulting vision probability of the scene. Viewing the scene as shown in Figure 7 the resulting perception is $P_{x,z} \cong 0.16$ while viewing as shown in Figure 8 yields $P_{x,z} \cong 0.09$; the parameters involved in the perception computations have the following values: $a = 16$ m, $l_a = 6$ m, $b = 20$ m, $l_b = 10$ m, and $c = 0.0064$ m⁻¹. The scene has a height of 6 m. Clearly the perception of the scene as viewed in Figure 7 is higher than Figure 8. This means that the probability of getting the visual information of objects present in the scene shown in Figure 7 is higher than for the objects in Figure 8.

Figure 7 Scene corresponding to the perception computation shown in Figure 6, where $a = 16$ m, $l_a = 6$ m, the height of the scene is 6 m and $c = 0.0064$ m⁻¹; the degree of perception of the scene is $P_{x,z} \cong 0.16$

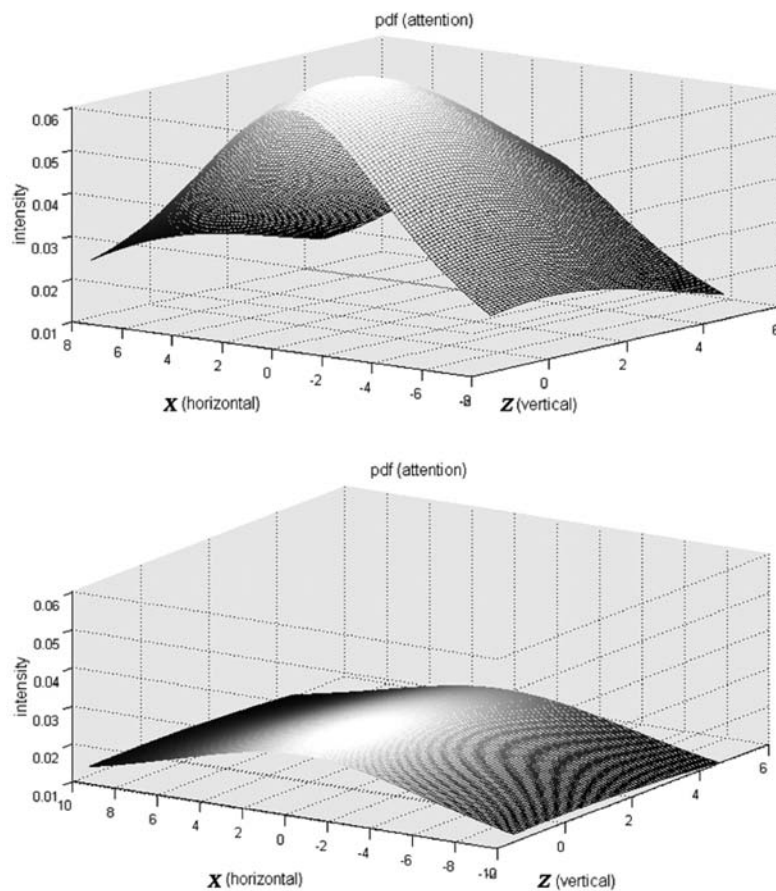


Figure 8 Scene corresponding to the perception computation shown in Figure 6, where $b = 20$ m, $l_b = 10$ m, the height of the scene is 6 m and $c = 0.0064$ m⁻¹; the perception of the scene is $P_{x,z} \cong 0.09$



The increase in perception is about factor 1.8 although the distance is only increasing by a factor of about 1.6. This indicates the relevance of distance on perception during early vision. To illustrate the difference the plot of the two-dimensional perception $P_{x,z}(x,z)$ along the x axis (horizontal) and z axis (vertical) is shown in Figure 9. The result obtained corroborates common perception experience: Viewing a scene from a closer distance we obtain more visual information. Since the difference in terms of unbiased vision probability is quantified, spaces can be precisely compared in architectural design. An architect can determine the degree of visual detail of a scene that should come to the awareness of a person during early vision by defining the spatial envelope accordingly. He/she can determine this perception degree using the model presented above. Occupants have a general demand for large spaces, which have a lower degree of perception with respect to their spatial envelope. The demand is presumably due to the fact that a high degree of perception of the spatial envelope is distracting during other mental activities. In other circumstances a high degree of perception of a scene is demanded, for example in case of a stadium or a concert hall. In these cases the positioning of the viewing locations takes this demand into account.

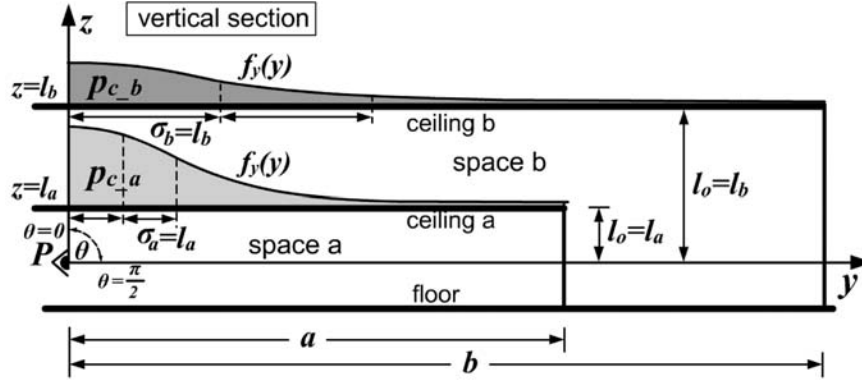
Figure 9 Plots of the two-dimensional pdf (attention) that belong to the scenes shown in Figures 7 and 8; the upper plot corresponds to Figure 7 where $l_o = l_a = 6$ m, and the lower plot to Figure 8, where $l_o = l_b = 10$ m



3.2 Perception of ceilings

Another example showing the significance of visual perception for architecture concerns the choice of ceiling height. Ceiling height has a significant influence on the perception of a ceiling itself. In the following the influence of ceiling height on perception during early vision is exemplified. Figure 10 shows a section view of two spaces labelled *space a* and *space b* with two different ceiling heights l_a and l_b as well as different lengths a and b .

Figure 10 Vertical section illustrating the effect of changing the ceiling height from l_a to l_b on the perception of the ceiling during early vision



The degree of perception of a ceiling differs depending on the distance the ceiling is viewed from and the size of the ceiling. In Figure 10 the respective probability density functions $f_y(y)$ that belong to the different ceiling heights are sketched. The early vision probabilities of *ceiling a*, denoted by p_{c_a} and of *ceiling b*, denoted by p_{c_b} are indicated by the shaded areas in the $f_y(y)$ vs y plots. The characteristic distance l_o is the distance between the ceiling and the eye height of the observer and denoted as $l_o = l_a$ and $l_o = l_b$ for the respective ceilings. To obtain the difference in degree of perception of the ceilings p_{c_a} and p_{c_b} are computed as follows. For the sake of clarity of the explanation first perception along the y -axis is compared, modelling the ceiling as a one-dimensional entity.

We take the scope of vision as $0 \leq \theta \leq \pi/2$, which yields:

$$f_y(y) = \frac{2}{\pi} \frac{l_o}{(l_o^2 + y^2)} \quad (0 \leq y \leq \infty) \quad (31)$$

as the probability density function of perception. For $l_o = l_a$, from Figure 10, the integral of attention yields:

$$p_{c_a} = \int_0^a f_y(y) dy = \frac{2l_a}{\pi} \int_0^a \frac{dy}{y^2 + l_a^2} = \frac{2}{\pi} \operatorname{arctg} \frac{a}{l_a} \quad (32)$$

For $l_o = l_b$, from Figure 10, the integral of attention yields:

$$p_{c_b} = \int_0^b f_y(y) dy = \frac{2l_b}{\pi} \int_0^b \frac{dy}{y^2 + l_b^2} = \frac{2}{\pi} \operatorname{arctg} \frac{b}{l_b} \quad (33)$$

As an illustrative example, assuming that $a/l_a = 10$ the early vision probability p_{c_a} becomes:

$$p_{c_a} = \frac{2}{\pi} \arctg 10 \cong \frac{2}{\pi} 1.47 \cong 0.94 \quad (34)$$

Assuming $b/l_b = 5$ the perception p_{c_b} becomes:

$$p_{c_b} = \frac{2}{\pi} \arctg 5 \cong \frac{2}{\pi} 1.37 \cong 0.87 \quad (35)$$

The results in Equations (34) and (35) show that the perception of *ceiling a*, denoted by p_{c_a} is higher than the perception of *ceiling b*, denoted by p_{c_b} .

The difference in perception is accentuated by the fact that the scene viewed is more accurately modelled as a two-dimensional surface, in contrast to the computation above, where the scene is modelled as one-dimensional. This is accomplished as follows. In the two-dimensional case perception during early vision is considered in forward and horizontal directions. For the forward direction the probability density along the y axis is considered as it is shown in Figure 10. Taking the visual scope as $0 \leq \theta \leq +\pi$, the corresponding attention probability density is:

$$f_y(y) = \frac{2}{\pi} \frac{l_o}{y^2 + l_o^2} \quad (36)$$

and an elemental perception during early vision is:

$$P_y = \int_{\Delta y} f_y(y) dy \cong f_y(y) \Delta y \quad (37)$$

For the horizontal direction the same geometry shown in Figure 10 applies also, but instead of the y -axis we consider perception along the x axis shown in Figure 1. As visual scope along the x -axis $-\pi/2 \leq \theta \leq \pi/2$ is taken, so that the corresponding attention probability becomes:

$$f_x(x) = \frac{1}{\pi} \frac{l_o}{x^2 + l_o^2} \quad (38)$$

and an elemental perception during early vision is:

$$P_x = \int_{\Delta x} f_x(x) dx \cong f_x(x) \Delta x \quad (39)$$

It is important to note that in the two-dimensional perception case the events of perceiving an object in the y -direction vs. the x -direction are *independent events*. Therefore, the joint probability of the vision probabilities is computed by means of multiplication of the probabilities belonging to each dimension, given by Equations (37) and (39). Explicitly we write:

$$P_{x,y}(x,y) = P_x P_y = \int_{\Delta x} \frac{1}{\pi} \frac{l_o}{x^2 + l_o^2} dx \int_{\Delta y} \frac{2}{\pi} \frac{l_o}{y^2 + l_o^2} dy \quad (40)$$

Please note that in Equation (40) dx and dy are infinitesimally small distance intervals around x and y . In the following perception comparison the joint probability given by Equation (40) is approximated by:

$$P_{x,y}(x,y) \cong \frac{2}{\pi^2} \frac{l_o^2}{(x^2 + l_o^2)(y^2 + l_o^2)} \Delta x \Delta y \quad (41)$$

taking Δx and Δy as sufficiently small distances.

The effect of ceiling height on the perception of a ceiling during early vision is exemplified in two scenes given in Figures 11 and 12. Figure 11 shows a space corresponding to *space a* in Figure 9, where $a = 15$ m and $l_a = 1.5$ m. Figure 12 shows a space corresponding to *space b* in Figure 9, where $b = 22$ m and $l_b = 4.4$ m. In both cases the ceiling is 18 m wide, i.e. it extends 18 m in the x -direction. In Figures 11 and 12 the degrees of elemental perceptions are given as an array of numbers. Each number gives the early vision probability belonging to a ceiling patch with the size $\Delta x \Delta y$ and the probability is plotted at the centre point of the respective patch. The summation of these elemental perceptions yields the resulting degree of perception of the ceiling. The perception degree of the ceiling shown in Figure 11 is $P_{x,y} \cong 0.94$, while it is $P_{x,y} \cong 0.50$ for the ceiling shown in Figure 12. It is important to note that the probabilities computed in the present case belong exclusively to the ceiling and not the entire space. This is done for appropriate comparison of the perceptions. Although the ceiling of the space shown in Figure 11 is smaller in size compared to the ceiling shown in Figure 12, the former has a significantly higher degree of perception compared to the ceiling of the latter space. The degree of perception of the ceiling in Figure 11 is about factor 1.8 higher compared to the ceiling in Figure 12. The meaning is that the ceiling shown in Figure 11 has much more visual impact compared to the ceiling shown in Figure 12. This is due to the difference in ceiling height.

To illustrate the difference the plot of the two-dimensional perception $P_{x,y}(x,y)$ along the axis x (horizontal) and y (forward) is shown in Figure 13. In the figure the location of the observation point is at $x = y = 0$.

From Figure 13 please note that the major contribution to perception during early vision stems from the region of ceiling nearby the observation point, where the visual attention is relatively high compared to distant regions. The perception of ceiling is an important issue in architecture as ceilings are most common spatial elements. Often it is undesirable that the ceiling has a high degree of perception. This is because generally the ceiling is not an environmental object to carry relevant visual information, but instead is a necessary element for other reasons. A ceiling with a high degree of perception or attention can be considered disturbing as it distracts from more relevant information in the environment. This issue becomes particularly noticeable in spaces with large dimensions, where the ceiling will have a substantial degree of perception unless it is placed at sufficient height. Architects know this by experience; however it was not possible to quantify the perceptual effect precisely and to compare different situations on a common ground.

Figure 11 Space corresponding to the perception computation shown in Figure 9, where $a = 15$ m, $l_a = 1.5$ m, and the extent of the ceiling in x -direction (horizontal) is 18 m; the resulting degree of perception of the ceiling is $P_{x,y} \cong 0.94$



Figure 12 Space corresponding to the perception computation shown in Figure 9, where $b = 22$ m and $l_b = 4.4$ m, and the extent of the ceiling in x -direction (horizontal) is 18 m; the resulting degree of perception of the ceiling is $P_{x,y} \cong 0.51$

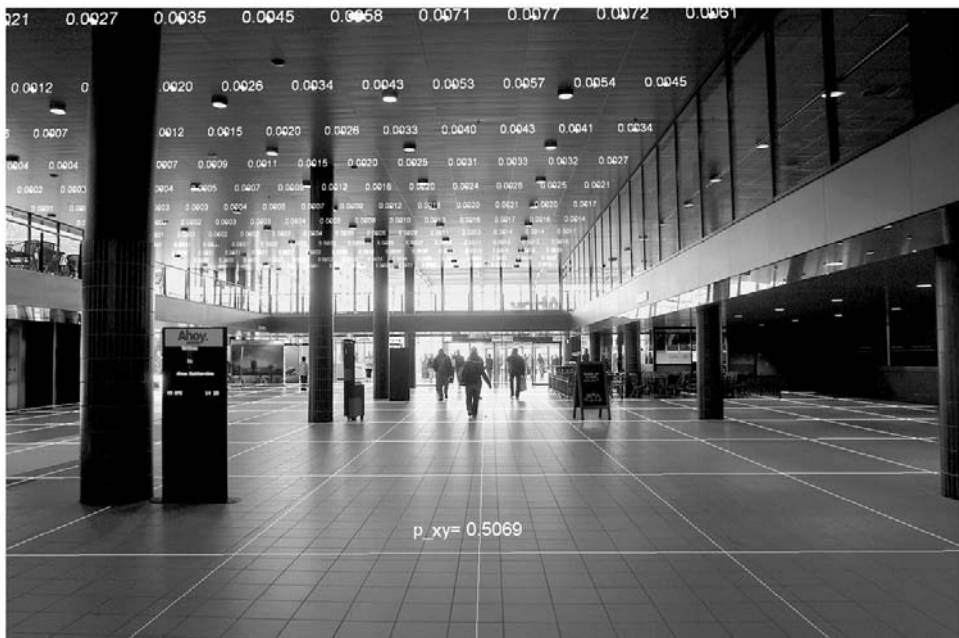
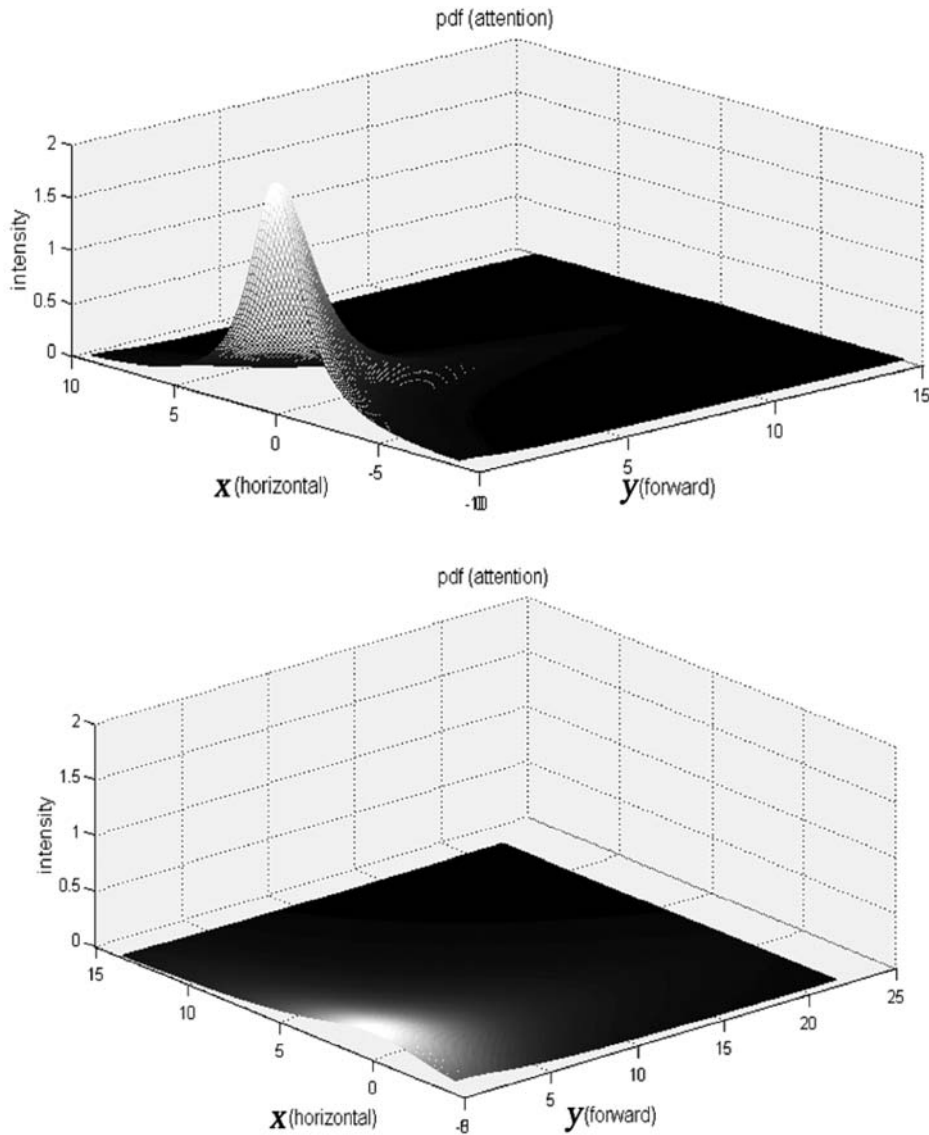


Figure 13 Plots of the pdf (attention) that belong to the ceilings shown in Figures 11 and 12; the upper plot corresponds to Figure 11, where $l_o = l_a = 1.5$ m and the lower plot to Figure 12, where $l_o = l_b = 4.4$ m



It is interesting to note that the attention pdf is well approximated by a Gaussian (Ciftcioglu et al., 2007), where the standard deviation σ is found to be $\sigma \approx l_o$. This implies $\sigma_a \cong l_a$ and $\sigma_b \cong l_b$, which is indicated in Figure 10. As the statistical properties of the Gaussian are well known, the relevance of attention/perception to architecture can be easily interpreted. Namely, having a distance l_o between viewpoint and ceiling, if we consider a region of the ceiling of length l_o in the y -direction shown in Figure 10, the early vision probability becomes about 0.68. In the same way, if we consider a region of length

$2l_o$ the vision probability becomes about 0.95. In the horizontal direction, if we consider a region of the length $2l_o$ in the y direction the vision probability becomes 0.68 (please refer to Figures 10 or 13). As a rule of thumb, for architects to calculate the vision probability of a ceiling we can consider a square shaped surface patch extending with the length $2l_o$ in the y -direction and $2l_o$ in the x -direction. The vision probability of the patch is approximately $P_{x,y} \cong 0.68 \times 0.95 \cong 0.65$. Considering a patch that extends with $2l_o$ in the y -direction and $4l_o$ in the x -direction, the perception of the patch is approximately $P_{x,y} \cong 0.95^2 \cong 0.90$.

4 Discussion and conclusion

In this work a model of early human vision is presented. Based on the model several vision-related concepts are mathematically described. Among these are attention and perception. Although these concepts are commonly exercised in daily life, the research illustrates how to gain insight into their relationships in a more definitive way. This is due to mathematical definitions of these concepts with minimal ambiguity. The validity of the model is confirmed, as the model outcome coincides with several manifestations we commonly experience: An observer is usually more aware of objects that are:

- located in the frontal region of our visual scope vs side regions
- that are nearby vs distant
- that are large vs small.

It is a basic common experience that these properties increase perception of an object. In this respect it is important to mention firstly that the corroboration of the theoretical results with daily experience is a significant indication of the validity of the theoretical considerations. Secondly, the results provide some insight into the mechanism responsible for these common phenomena. Thirdly, the relevance of distance to perception is quantified, as opposed to a vague verbal statement, such as that ‘perception decreases with distance’. We intuitively know that the closer an object is located to us the more aware we are of it. However, in a general case, reducing the distance of an object by half does not increase perception by a factor of two per space dimension, in contrast to what one may assume. This means that the amount of visual detail obtained from an object does not increase proportionally to the viewing distance in general. The relation is generally non-linear and subject to computation. Precise comprehension of human vision in mathematical terms is desirable to convert it directly to engineering systems or to apply it in design. This research is especially relevant in architectural design, where it is a significant step towards understanding the effects of geometric modifications with greater precision and to reflect that in the design. The vision-related concepts are defined in probabilistic terms, so that the bio-induced complexity, imprecision and vagueness may be absorbed. The complexity stems from the brain processes, which are ultimately effective for cognition. In the present model, attention is defined as a probability density. The integration of this attention over a spatial domain is perception. As these concepts are merely verbally defined in the literature the definition presented in the work is novel and subject to adoption into diverse fields.

It may be of value to point out that the numeric result from the computation of early vision probability is equivalent to the solid angle of an object in the visual scope normalised by the angle of the scope. Previously, J.J. Gibson conceived his *ambient optic array*, a set of solid angles (1986). However, of course, the calculation of a solid angle that is normalised by the angle of the visual scope by itself is an ad hoc proposition as a model of perception. For example, it is difficult to justify *a priori* why distance between object and observer or the orientation of an object would not matter. But, more importantly, it is unclear what the resulting number actually means. So the major contribution of the model presented in this work is that it defines perception during early vision as a *probability* with the associated pdf; namely the probability that the corresponding environmental information is mentally realised. The validity to take the normalised solid angle as measure of perception is confirmed by the theoretical considerations behind the present work, which is based on its premise and further derivations. The two alternative ways to calculate perception have some differences. One difference is that the solid angle approach merely informs about the vision probability of an object in the form of a scalar number, whereas the probabilistic approach involves the concept of *attention*, which is a probability density and given per unit length and in a certain direction. Therefore, the orientation of the surface of an object is neglected in the solid angle computation as well as its distance, while in the probabilistic case this information is expressed by a probability density in space. This means the solid angle model is useful to compute the vision probability of an object, if one is not interested how the perception density is varying throughout the visual scope or within the boundaries of an object. Particularly, the directions from the perspective of an observer, which are more prominent in the perception sense, are given by the pdf and not by the probability. In architectural design, the variation of perception density is interesting information, because in the design of a space which locations in the space receive more or less attention or which directions are most prominent may be relevant. Certainly the probability density function, i.e. attention, may be approximated by subdividing the object into smaller patches and calculating their respective normalised solid angles. However, this may require ample computation depending on the accuracy needed in discrete form, yet it does not provide a clue about the analytical form or origin of the pdf in question. In the analytic case, for any complex vision model the formulation in principle remains the same. In later phases of vision, or for any other reason, a non-uniform pdf may be used as a starting point in the computation of the early vision probability using the same theoretical probability considerations to arrive at the result. Such a situation occurs as a biased case after early vision; namely, the early vision probability is further elaborated by using the attention pdf as a bias, and with the presence of this bias one can investigate the perception in this new phase. Such a situation occurs if one views away a scene beyond the early vision phase.

Provided that a spatial design fulfils the basic functional requirements, its architectural quality and thereby part of the value of the building are also dependent on attention and perception issues. In the present work, the perception model is applied to compare scenes with respect to attention and vision probability, i.e. perception. By quantifying the influence of geometry on perception in this vision phase the model provides a starting point, so that perception can be treated in design with great awareness and on a common ground. The opportunity to express the perception via a probability may become particularly helpful when an architect needs to verify multiple perceptual requirements simultaneously, while he/she is only able to verify the perceptual aspects from one

viewpoint at a time. This is the case, for instance, in the design of a housing complex or a larger retail interior.

Ceiling height and distance to a scene are important factors determining perception. In this work their influence is quantified, so that the ceiling height or the size of a space can be selected for the intended perceptual impact. Perception plays a role in the quality of life. In residential or office architecture, low visual perception is quite commonly required. This is a reason why large spaces, which generally have a lower degree of perception along their spatial envelope, are in demand. Requirements for low perception presumably stem from the fact that a high degree of perception of the environment entails a mental effort that may conflict with other mental activities. This explains why the visual experience from a mountain-top, or the view of a distant landscape may be considered 'uplifting' or 'freeing the mind'. In other situations a high degree of perception of a scene is demanded. This applies for example in case of a stadium or a concert hall.

Visual perception involves many factors that are related to the personal memory of an observer. To be able to address perception accordingly, it is useful for an architect to have some impartial information about the early vision probability of the objects in the environment. The perception model presented in this work is unbiased with respect to the viewing direction, so that conditional preferences of an observer are not included in the outcome of the model. This may be desirable in situations where users of the space are not known in advance, e.g. in the model all directions in the visual scope are treated as being equivalent, so that the influence of geometry on perception is modelled uniquely. This is a beneficial feature concerning the application of the model in design: An architect can take the result of the geometric early vision model as an unbiased base for the early design. Based on the results architects can further 'shape' the vision probability by using means such as increasing colour, or texture contrast. In this way an architect may increase the perception of an object, which he/she deems insufficient. Or, alternatively he/she may try to reduce the perception of an object deemed to have too much vision probability.

It is important to note that the geometry of an object almost always has an effect on its perception. Exceptions are, for example, when there is no light to see anything, or if, more generally, there is insufficient colour contrast among the object and its background. In these cases, the early vision probability of the object vanishes. Please note that the present model of perception, where the parameters of the model are exclusively geometric, can be taken as a model of perception under the condition that the colour differences among the objects and their respective backgrounds are about the same for all objects. To account for the influence of colour differences it is an interesting relevance to combine the geometric perception model with a model describing the influence of colour during early vision. Next to colour, other features of an object may influence its vision probability, e.g. texture contrast. Colour contrast, texture contrast, etc. may increase or decrease the early vision probability that is due to geometry. Models of these features that quantify the respective influence on the seeing probability can be integrated to the present geometry-based model. In perception experiments to identify such an influence, without information on the role of geometry in perception, which component of a vision probability is due to geometry, and which component stems from other factors would not be identifiable. In this sense the present perception model for early vision reduces the complexity of the modelling task, so that detailed insight into the influence of other aspects may be obtained.

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