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On the Efficiency of Multivariable TS Fuzzy Modeling

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***Abstract*— Studies on the efficiency of multivariable Takagi-Sugeno fuzzy modeling is described. From the interpretability viewpoint fuzzy modeling becomes complex especially in multidimensional cases due to decomposition of multidimensional fuzzy sets for each dimension. Even in one dimensional case, appropriate selection of fuzzy sets in number and shape is essential for satisfactory models. This general observation in mind, this research aims to make a study on the efficiency of fuzzy modeling taking two exemplary cases. Both cases are considered to be representing a dynamic system in two dimensional space. They are distinguished by the degree of nonlinearity they have. Both systems are modeled by fuzzy logic and the outcomes are comparatively studied to show explicitly how the modeling performance varies being dependent on the degree of nonlinearity involved.**

I. Introduction

THERE are a variety of approaches to modeling of complex systems. Fuzzy modeling plays an important role among these, since it is based on fuzzy logic which is one of the essential paradigms of computational intelligence. Fuzzy modeling is widely treated in the literature and from the reported applications, one notes that it functions satisfactorily for the intended goals. Fuzzy models are rather transparent to interpretation and analysis. However, as to variety of fuzzy modeling approaches, absence of a clear procedure for general fuzzy modeling already is a firm indication of the complexity of the task whose subject-matter is to provide transparency. Among the issues present in this context one can refer to the shapes, locations and number of the fuzzy membership functions, at the first instance. Nevertheless, for certain type of fuzzy modeling rather well-established methods are available for practical applications. There are two distinctive fuzzy modeling paradigms. The one is known as Mamdani-type [1,2] where linguistic relations are represented as constant fuzzy sets at the antecedent and consequent spaces. The other one is known as Takagi-Sugeno (TS) model [3] where locally linear sub-models are used at the consequent spaces. Since, in most cases the knowledge about the system being modeling is not enough to place the fuzzy sets and consequent parameters, data driven models are ubiquitously used. For this purpose least square estimation from the observation data is commonly considered. In determining the antecedent fuzzy sets, in the multivariable input case, one clusters the data to obtain multivariable fuzzy sets and by means of projection on each input variable one obtains the respective fuzzy sets. However, during the projection process an irrecoverable information loss occurs so that, the fuzzy modeling from the projected fuzzy sets becomes an issue. As to one-dimensional input case, there is no need for projection and therefore the model is satisfactorily accurate. This explains the reason why the majority of fuzzy modeling applications concern merely this simple case. However, in a low-dimensional case, the model becomes less precise and the quality of the model exponentially aggravated with the increase of the number of fuzzy sets employed in each dimension. Note that this is not only because of curse of dimensionality but also because of information loss due to decomposition of the multivariable fuzzy sets. Although the fuzzy modeling research and applications are extensive and apparently model performance is satisfactory from the reported results, to author's best knowledge there is no much work on making comparison to demonstrate the effectiveness of fuzzy modeling depending on the nonlinearity it models. The nonlinearity is one of the important measures of complexity. This work aims to highlight above-mentioned issue, namely the efficiency of fuzzy modeling as to nonlinearity by comparison of results obtained from two different simulated nonlinear dynamic systems. The motivation of this paper is to investigate the nonlinearity performance of fuzzy modeling, rather than to report results of another fuzzy modeling application of a nonlinear system, as the latter is the general trend among the reported works.

The organization of the paper is as follows. It describes briefly the foundations of fuzzy modeling and defines also two nonlinear functions with different degree of nonlinearities. By means of these, the interpolation performance of a fuzzy model is investigated whilst the approximation performance is also investigated by projected and non-projected multivariable membership functions. These model issues are interdependent and all play role on the model outcome and they are investigated by means of computer experiments. The comparative results are reported in detail pointing out their salient implications.

II. Fuzzy Modeling

A. Takagi-Sugeno Fuzzy Modeling

Takagi-Sugeno (TS) type fuzzy modeling [3] consists of set of fuzzy rules a local input-output relation in a linear form as

$$R_i : If \quad x_1 \quad is \quad A_{i1} \quad and \ldots x_n \quad is \quad A_{in} \\ Then \quad \hat{y}_i = a_i x + b_i \,, \qquad i = 1,2,\ldots\ldots,K \qquad (1)$$

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where R_i is the ith rule, $\mathbf{x}=[x_1,x_2,\ldots,x_n]^T \in X$ is the vector of input variables; $A_{i1}, A_{i2},\ldots,A_{in}$ are fuzzy sets and y_i is the rule output; K is the number of rules. The output of the model is calculated through the weighted average of the rule consequents of the form

$$\hat{y} = \frac{\sum_{i=1}^{K} \beta_i(x)\hat{y}_i}{\sum_{i=1}^{K} \beta_i(x)} \quad (2)$$

In (2), $\beta_i(x)$ is the degree of activation of the i-th rule

$$\beta_i(x) = \prod_{j=1}^{n} \mu_{A_{ij}}(x_j), \quad i = 1,2,\ldots,K \quad (3)$$

where $\mu_{A_{ij}}(x_j)$ is the membership function of the fuzzy set A_{ij} at the input (antecedent) of R_i. To form the fuzzy system model from the data set with N data samples, given by

$X=[\mathbf{x}_1, \mathbf{x}_2, \ldots,\mathbf{x}_N]^T$, $Y=[y_1, y_2, \ldots, y_N]^T$ (4)

where each data sample has a dimension of n (N>>n). First the structure is determined and afterwards the parameters of the structure are identified. The number of rules characterizes the structure of a fuzzy system. The number of rules is determined by clustering methods. Fuzzy clustering in the Cartesian product-space X×Y is applied to partition the training data. The partitions correspond to the characteristic regions where the system's behaviour is approximated by local linear models in the multidimensional space.

Given the training data Γ and the number of clusters K, a suitable clustering algorithm [4] is applied. One of such clustering algorithms is known as Gustafson-Kessel (GK) [5]. As result of the clustering process a fuzzy partition matrix U is obtained. The fuzzy sets in the antecedent of the rules are identified by means of the partition matrix U which has dimensions [N×K]; N is the size of the data set. The *ik*-th element of $\mu_{ik} \in [0,1]$ is the membership degree of the i*th* data item in cluster k; that is, the i*th* row of U contains the point wise description of a multidimensional fuzzy set. One-dimensional fuzzy sets A_{ij} are obtained from the multidimensional fuzzy sets by projections onto the space of the input variables x_j. This is expressed by the point-wise projection operator of the form [6,7]

$$\mu_{A_{ij}}(x_{jk}) = proj_j(\mu_{ik}) \quad (5)$$

The point-wise defined fuzzy sets A_{ij} are then approximated by appropriate parametric functions. The consequent parameters for each rule are obtained by means of linear least square estimation. For this, consider the matrices

$X=[\mathbf{x}_1,\ldots, \mathbf{x}_N]^T$, $X_e[X,\mathbf{1}]$ (extended matrix [N×(n+1)]) ; Λ_i (diagonal matrix dimension of [N×N]) and

$X_E=[(\Lambda_1 X_e); (\Lambda_2 X_e);\ldots (\Lambda_K X_e)]$ ([N×K(n+1)]) (6)

where the diagonal matrix Λ_i consists of normalized membership degree as its k-th diagonal element

$$_n\beta_i(x_k) = \frac{\beta_i(x_k)}{\sum_{j=1}^{K} \beta_j(x_k)} \quad (7)$$

The parameter vector ϑ dimension of [K×(n+1)] is given by

$\vartheta=[\vartheta_1^T \; \vartheta_2^T \; \ldots\ldots \; \vartheta_K^T]^T$ (8)

where $\vartheta_i^T=[a_i^T \; b_i]$ $(1 \le i \le K)$. If we denote the input and output data sets as X_E and Y respectively then, the fuzzy system can be represented as a regression model of the matrix form

$Y=X_E\,\vartheta+e$ (9)

For a model with single output (9) becomes

$y=X\,\vartheta+e$ (10)

where

$\vartheta^T=[a^T \; b]$ (11)

Due to projection process in (5), there is an information loss of multidimensional fuzzy sets which is reflected as degradation in the fuzzy modeling. This is referred to as *reconstruction error* and depicted schematically as seen in figure 1.

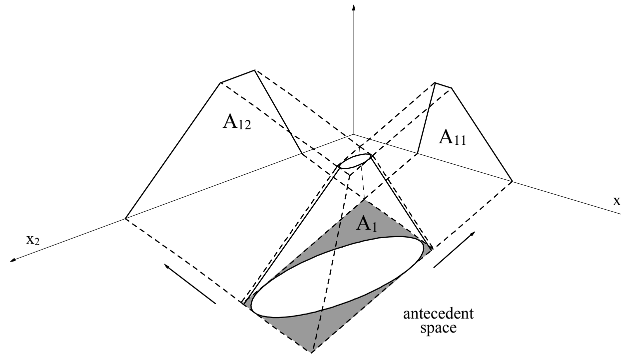

Fig. 1. Schematic representation of error due to reconstruction from projected membership functions to multidimensional membership functions.

To circumvent the reconstruction error, the multidimensional fuzzy sets are directly used for modeling without decomposition through projections.

B GK Clustering Algorithm

For clustering, there are several effective fuzzy clustering algorithms available. Gustafson-Kessel algorithm is the one, which is commonly used due to some desirable features. In order to detail the modeling enhancement achieved in this work, GK algorithm is briefly explained below.

GK Algorithm: Given the data Z with the number of data samples N, number of clusters M, fuzziness parameter m>1, and the termination tolerance ε>0, initialize the fuzzy partition matrix $U=[u_i]$(i=1,2,..M), randomly. Then

Repeat for l=1,2,...

Step1: Compute cluster prototypes

$$v_i^{(l)} \frac{\sum_{k=1}^{N} (u_{ki}^{(l-1)})^m z_k}{\sum_{k=1}^{N} (u_{ki}^{(l-1)})^m}, \quad 1 \le i \le M \quad (12)$$

Step 2: Compute covariance matrices:

$$F_i = \frac{\sum_{k=1}^{N} (u_{ki}^{(l-1)})^m (z_k - v_i^{(l)})(z_k - v_i^{(l)})^T}{\sum_{k=1}^{N} (u_{ki}^{(l-1)})^m}, 1 \le i \le M \quad (13)$$

Step 3: Compute distances to cluster prototypes:

$$d_{ki}^2 = (z_k - v_i^{(l)})^T D_i (z_k - v_i^{(l)}), 1 \le i \le M, 1 \le k \le N \quad (14)$$

where

$$D_i = [\det(F_i)^{1/(n+1)} F_i^{-1}] \quad (15)$$

which is called *norm-inducing matrix*.

Step 4: Update the partition matrix:

$$\begin{aligned} &for\ \ 1 \le i \le M, 1 \le k \le N \\ &if\ \ d_{ki} > 0 \\ &u_{ki}^{(l)} = \frac{1}{\sum_{j=1}^{M} (d_{ki}/d_{kj})^{2/(m-1)}} \\ &else\ \ if\ \ d_{ki} = 0 \\ &u_{ki}^{(l)} = 1 \end{aligned} \quad (16)$$

also,

$$u_{ik}^{(l)} \in [0,1]\ \ with\ \sum_{i=1}^{M} u_{ik}^{(l)} = 1 \quad (17)$$

until $\| U^{(l)} - U^{(l-1)} \| < \varepsilon$

Note that the product $d_{ki}^2 = (z_k - v_i^{(l)})^T D_i (z_k - v_i^{(l)})$ measures the distance of the antecedent vector z from the projection of the cluster centre $v_i^{(l)}$. In step four, an inversion is applied to obtain the membership degree where the expression computes the degree of fulfilment of one rule relative to the other rules and the sum of the membership degrees of all the rules equal to one. This is, in fact, is a probabilistic constraint. A drawback of the GK algorithm is that it is effective to find clusters of approximately equal volumes. However, the eigenstructure of the cluster covariance matrix provides information about the shape and orientation of the cluster. This information can be used to compute optimal local linear models from the covariance matrix, as this is exploited in this work for comparison purpose, comparing the common approach with the approach supported by RBF network in this work. A desirable feature of the GK algorithm over other fuzzy clustering methods is that GK can detect clusters of different shape and orientation in one data set, yielding better positioning of the fuzzy sets projected on the individual antecedent variables in a fuzzy model. As to fuzzy modeling with multivariate membership functions, the degree of fire (DOF) values for each variable can be obtained via norm inducing matrix and cluster centers information via (14). However in this case, since the membership function values are normalized after the computation, the normalization factor for each cluster is different than the normalization factor used for obtaining the fuzzy model. In other words, new conditions are imposed on the formation of the DOF determination rather than keeping the modeling conditions fixed. This situation introduces another source of error similar to the reconstruction error, which occurs in the case of decomposed multivariable fuzzy membership functions via projection. To avoid this, the multivariable membership functions are fixed by means of a network, which is radial basis function (RBF) network, in this case. By doing so, the modeling conditions are kept constant and DOF is obtained simply as the output of this network.

III. EXPERIMENTAL STUDY OF FUZZY MODELING

For the experimental study, modeling of a two input nonlinear functions

$$f_1(x_1, x_2) = 0.2 * (((x_2 - x_1^{\ 2})^2 - (1 - x_1))^2 - x_1^{\ 2} \quad (18)$$

and

$$f_2(x_1, x_2) = 0.2\left((x_2 - x_1)^2 - (1 - x_1)^2\right) \quad (19)$$

are considered as unknown dynamic systems. We can assume that two unknown dynamic systems are given in a general form $y(t) = F(x_1(t), x_2(t))$ which is subject to fuzzy modeling by means of two data sets with appropriate number of data samples given. The systems are a priory unknown for each case. The functions $f_1(x_1, x_2)$, $f_2(x_1, x_2)$ defining the surfaces are given by the data sets each of which contains 121 data samples. These functions are shown in figure 2 as upper and lower plots, respectively where apparently the function $f_1(x)$ has relatively higher nonlinearity. Each surface in figure 2 is subjected to fuzzy modeling by turn separately for comparison purpose. By this work it is intended to demonstrate that the fuzzy modelling performance is dependent on the degree of nonlinearity of the dynamic system. The source of different modelling performance stems from the reconstruction error as depicted in figure 1. To realize clearly the issue being introduced in this work, it should be clear that, if the fuzzy modeling is accomplished with the multidimensional fuzzy sets directly as obtained from the Gustafson-Kessel algorithm, then the model errors would be relatively less compared to the case if the multidimensional sets are projected by turn on each one-dimensional input-variable antecedent space and the model is formed via these projected sets. However, in one-dimensional case the projected fuzzy sets can be shaped in an analytical form as triangles or Gaussian or sigmoid, so that in actual utilisation of the model there is no interpolation problem since the fuzzy set is clearly defined. However in multidimensional case, since the fuzzy sets are not determined in an analytical form, in actual utilisation an appropriate interpolation method should be used to interpolate between the *modeling-data-samples* to determine the exact points on the multidimensional sets corresponding to the *test-data-samples.* The issue being presented is rather elusive since the fuzzy modeling procedure is generally rather straightforward and theoretically is well founded. Therefore, some subtle issues are deemed to be not significant. However, in actual implementation, such issues prove to be significant, depending on the application. The research being presented is an attempt to show that the model performance is dependent on the degree of the nonlinearity of the model and the issue mentioned above has a significant role on the performance quality. For this aim, first, the dynamic system given by $f_1(x)$ is fuzzy modelled. The locations of the multivariable membership functions are found by means of clustering and they are shown in figure 3 on the contour map as well as on the 3D surface. The adequate number of clusters for this model is identified to be five. Based on these multivariable membership functions the identified surface and the corresponding data points are given in figure 4. The identification of the surface is rather satisfactory in this case and this is also observed in figure 4 at the lowermost plot. Here the data points given are shown together with the identified data points and they do not differ significantly. The projected membership functions of the variables x_1 and x_2 based on the multivariable membership functions are shown in figure 5. The same surface identified by the projected membership functions of the variables x_1 and x_2 is shown in figure 6 (upper plot), where also the identified data points together with the given data points are shown (lower plot). In figure 6, it is clear that some data points are not matching the identified surface and the surface reconstruction is not satisfactory. To illustrate this more explicitly, the same model is used for testing employing 144 and 169 new data points and the estimated data points are shown in figure 7.

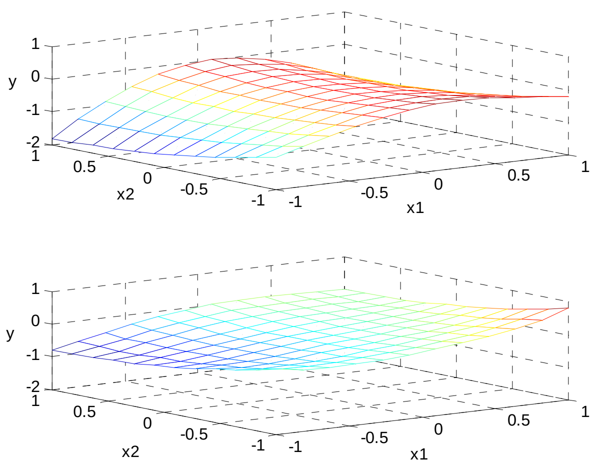

Fig. 2. Surfaces representing two nonlinear dynamical systems with different degree of nonlinearity.

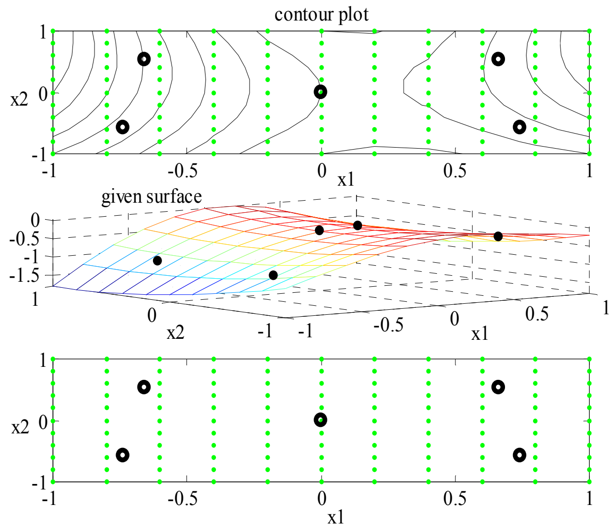

Fig. 3. GK clustering of the data with 121 data samples. Contour plot (uppermost), surface (middle) and data samples and the clusters projected on x1,x2 plane.

Clearly, these new points are estimated by interpolation and they altogether form the surfaces. The fuzzy modeling errors, even in such 2-dimensional nonlinear case are clearly observed. The source of the errors is attributed mainly to the reconstruction error, which is caused by the projection of the multivariable membership functions. The projected membership functions are formed by different Gaussians combined in such a way that the desired shape of the membership functions are obtained optimally in the least square sense.

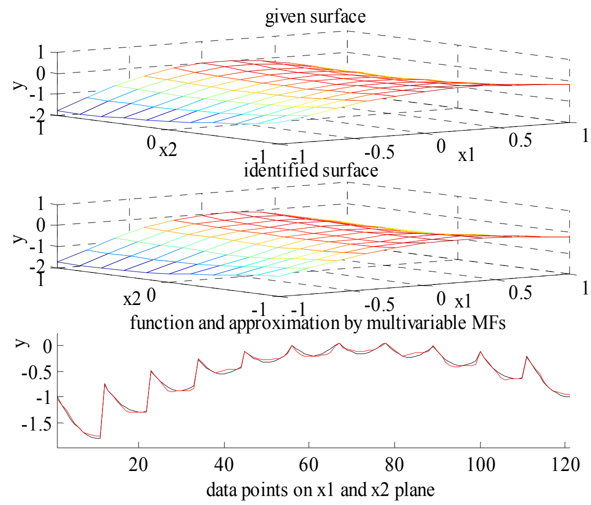

Fig. 4. Given and identified surfaces (upper plots) and the data points(lower plot) by five multivariable fuzzy membership functions.

Clearly, these new points are estimated by interpolation and they altogether form the surfaces. The fuzzy modeling errors, even in such 2-dimensional nonlinear case are clearly observed. The source of the errors is attributed mainly to the reconstruction error, which is caused by the projection of the

multivariable membership functions. The projected membership functions are formed by different Gaussians combined in such a way that the desired shape of the membership functions are obtained optimally in the least square sense.

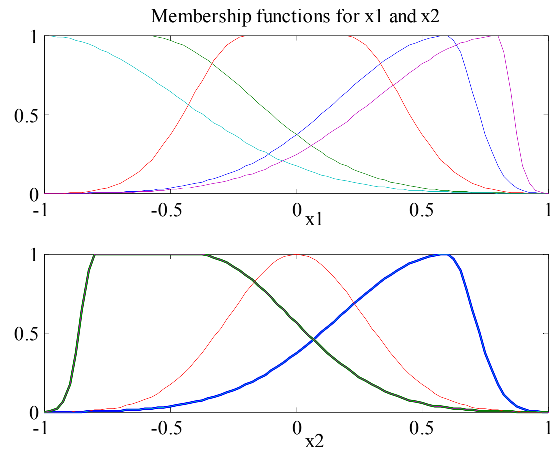

Fig. 5. Multivariable fuzzy membership functions after projection corresponding the clusters in figure 2.

The same identification task is carried out by means of multivariable membership functions using RBF network for interpolation. The results are shown in figure 8, for 121-points given and identified (upper); 144 and 169 points estimated by interpolation of the multivariable fuzzy membership functions. Note that, the results for 121 points are already presented in figure 4 (lowermost). The results for 144 and 169 are accomplished by an RBF network representing the fuzzy membership functions for each case. By the comparison of figure 6 and 7 with figure 8, the improvement achieved by using multivariable membership functions without projection is clearly seen.

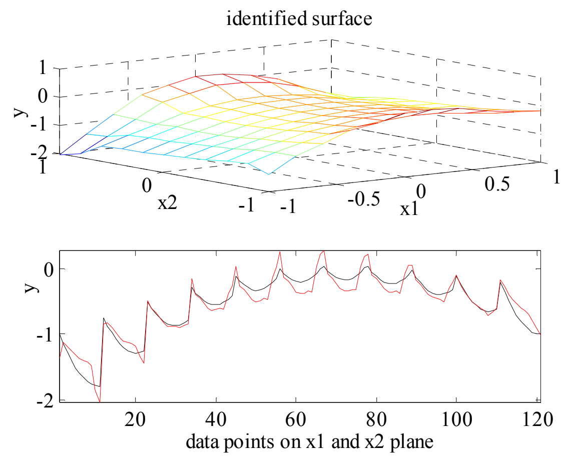

Fig. 6. Identified surface by projected membership functions (upper), and the identified data samples together with the given ones (lower).

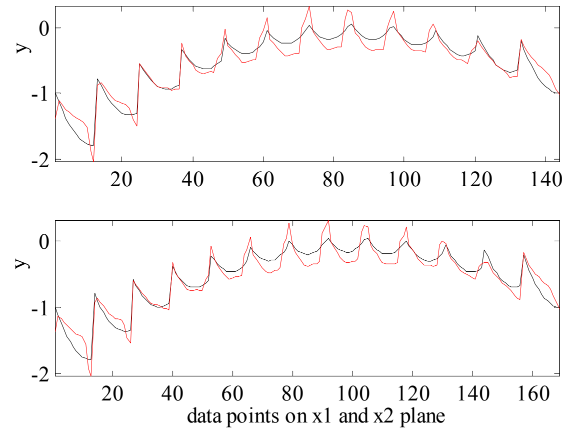

Fig. 7. Identified data points of f1(x1,x2) by five projected membership functions for 144 points (upper), and the same for 169 points (lower)together with the data points given.

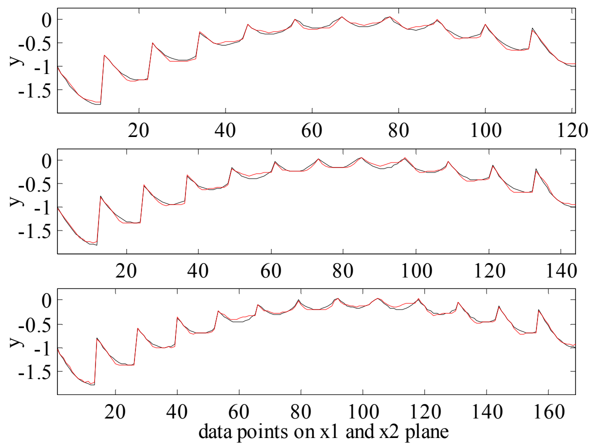

Fig. 8. The data points both given and identified by multivariable membership functions for121 points (upper); interpolation by RBF for 144 points(middle) and 169 points (lower).

In the case of forming multivariable fuzzy membership functions in place of RBF network, one can use the *norm inducing matrix*

$$D_i = [\det(F_i)^{1/(n+1)} F_i^{-1}] \tag{20}$$

and computes the membership function by

$$u_{ki}(x_k) = \frac{1}{\sum_{j=1}^{M} (d_{ki} / d_{kj})^{2/(m-1)}} \tag{21}$$

This expression is the degree of fulfilment of one rule relative to the other rules and the sum of the membership degrees of all the rules equals 1 as to fuzzy clustering. Because of the nature of the constraint involved, this is referred to as *probabilistic* method. Based on this method one obtains the results as presented in figure 9. Aiming to determining model efficiency as to nonlinearity, in the second step, the dynamic system given by $f_2(x_1,x_2)$ is fuzzy modeled. The locations of the multivariable membership functions are found by means of clustering and the clusters are shown in figure 10 on the contour map as well as on the surface. Here, to apply the same procedure as before to form multivariable fuzzy membership functions with 144 and 169 points, the norm inducing matrix with probabilistic approach is used. Also, the number of clusters for this model is taken as five as before for comparison. The projected fuzzy membership functions are shown in figure 11. In figure 12, the model efficiency is apparently satisfactory. This is to compare with figure 6, where the same model efficiency there is not satisfactory. One can conclude that, small increase of nonlinearity may affect the fuzzy model performance significantly.

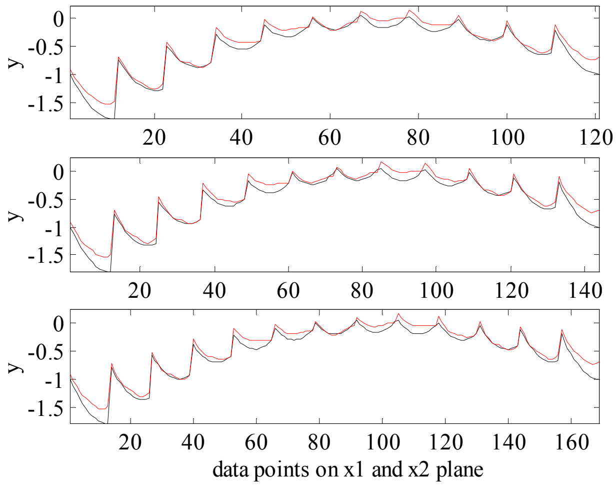

Fig. 9. Given and identified surfaces (upper plots) and the data points(lower plot) by five multivariable fuzzy membership functions. (upper); interpolation by norm-inducing matrix for 144 points(middle) and 169 points (lower).

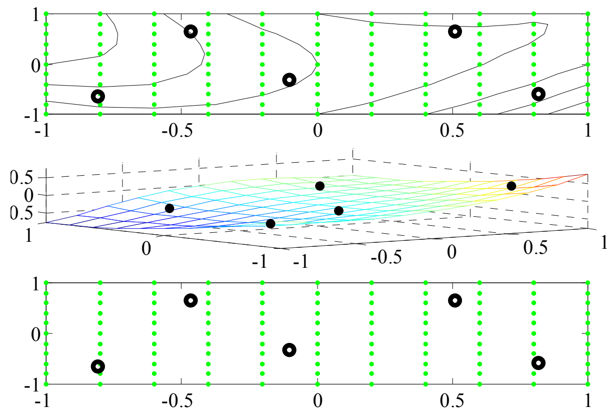

Fig.10. Given and identified surfaces (upper plots) and the data points(lower plot) by five multivariable fuzzy membership functions.

Using the projected membership functions the interpolation results with for 144 and 169 data points are presented in figure 13 which is to compare with figure 7. The comparison of figure 7 for $f_1(x_1,x_2)$ with figure 13 for $f_2(x_1,x_2)$ reveals that the reduction of nonlinearity provides improved fuzzy modeling performance as the same is noted by the comparison of figure 6 and 12. The $f_2(x_1,x_2)$ surface identified by the projected membership functions of the variables x_1 and x_2 is shown in figure 12 (upper plot) where also the identified data points together with the given data points are shown (lower plot).

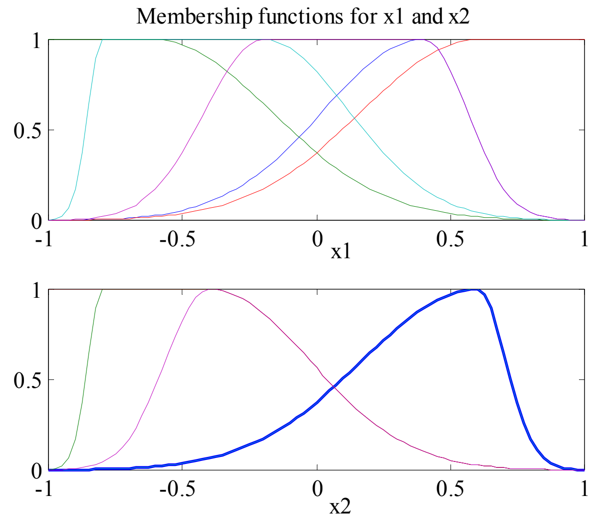

Fig. 11. Multivariable fuzzy membership functions after projection corresponding the clusters in figure 10.

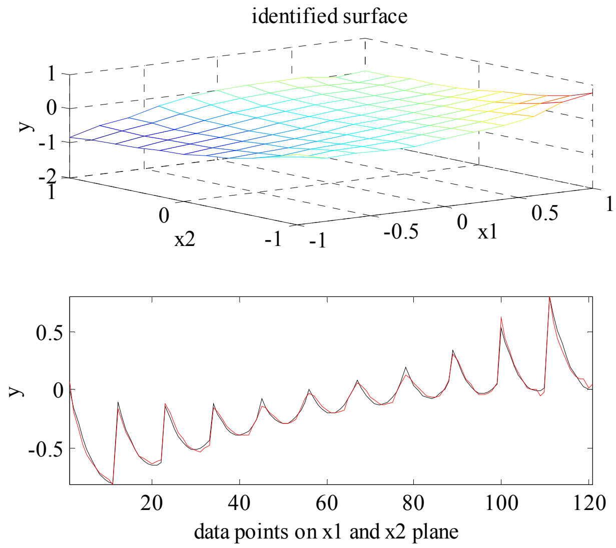

Fig. 12. Identified surface by five projected membership functions (upper), and the identified data samples together with the given ones (lower). A satisfactory coincide is observed.

The same identification task is carried out by means of multivariable membership functions with RBF network for interpolation. The results are shown in figure 14, for 121 points given and identified (upper), for 144 and 169 points by interpolation. The multivariable fuzzy membership functions are modeled by RBF network. Figure 14 is to compare with the figure 8. From the multivariable membership functions modeled by an RBF viewpoint, there is no significant fuzzy modeling performance difference. However, a slight improvement in the latter case is still observable by close inspection.

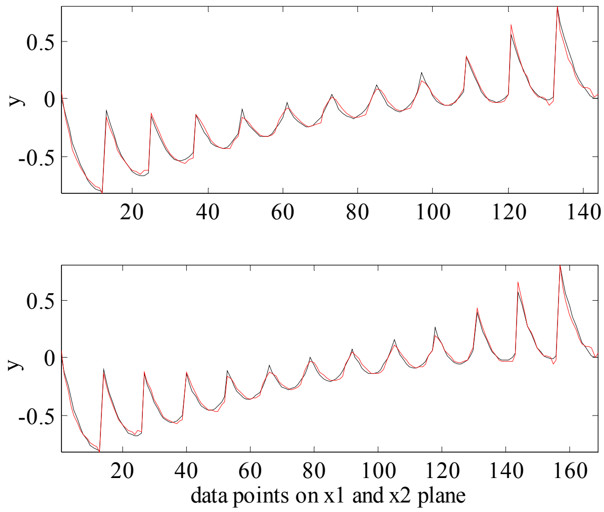

Fig. 13. Identified data points of *f2(x1,x2)* by five projected membership functions for 144 points (upper), and the same for 169 points (lower)together with the data points given.

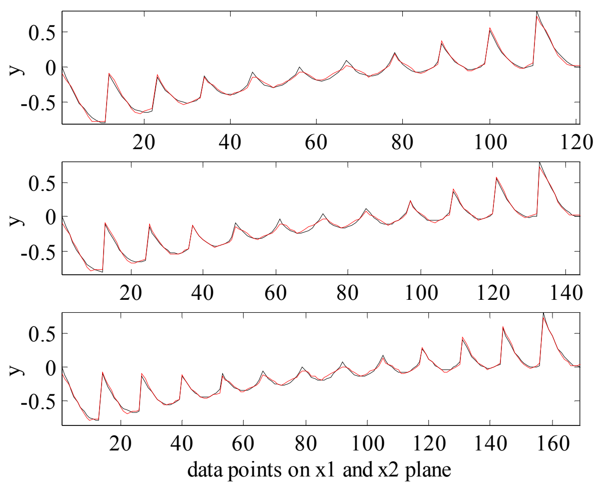

Fig. 14. The data points of *f2(x1,x2)* both given and identified by multivariable membership functions for 121 points (upper); interpolation by RBF for 144 points (middle) and 169 points (lower).

In the case of forming multivariable fuzzy membership functions, using the norm inducing matrix in place of using RBF network and the computing the membership function by (21), one obtains the results shown in figure 15 which is to compare with figure 9. The improvement in the latter case is clear where the sensitivity of fuzzy modeling to nonlinearity is also obvious. However, the approximation to the dynamics of an unknown nonlinear system by fuzzy modeling is still an important asset of the fuzzy methodology. Comparison of figures 14 and 15 indicate no salient difference between them although figure 14 approximates the surface of nonlinear dynamic system better due to additional RBF support for membership function modeling.

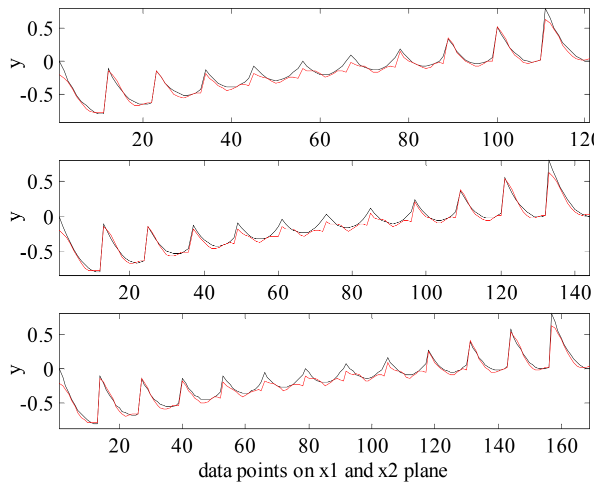

Fig. 15. The data points of *f2(x1,x2)* both given and identified by multivariable membership functions for 121 points (upper); interpolation by norm-inducing matrix for 144 points (middle) and 169 points (lower).

The comparative results presented through figures 2-15 are rather informative from the fuzzy modeling viewpoint. In the first place, comparison of figures 8 and 14 or figure 9 and 15 reveals that, the employment of multivariable fuzzy sets without projection provides more accurate modeling. Secondly, the accuracy can still be improved by integrating an RBF network for the interpolation of the multidimensional fuzzy sets. By comparison of figure 7 with the figure 8 or 9 it is clear that multivariable fuzzy sets are better not be projected on the antecedent space variables. The deviations seen in figure 6 are due to the projection errors. It is noteworthy to mention that the model performance is satisfactorily stable for test data sets with varying number of data samples, implying satisfactory performance of the model. Note that, the data samples in the test data sets are entirely different, that is, the sets with lower number of data samples are not the subsets of the sets with higher number of data samples. However, the performance sensitivity of a multivariable fuzzy model to nonlinearity of the system subject to modeling is clearly shown. Therefore, the fuzzy model should receive proper attention for its approximation satisfaction in the applications. Because of this very reason, the working fuzzy modeling implementations reported are generally restricted to one dimension or simple systems with limited complexity. Such a reported dynamic system surface is shown in figure 16 [8]. The satisfactory approximation of this surface with fuzzy modeling requires the special membership functions and fuzzy rule bases though the system is apparently not to deem to be complex being highly nonlinear. Although one can argue that, fuzzy model of a multidimensional system can be treated by a number of one-dimensional fuzzy models each of which is for one variable the reconstruction error due to projection can be quite significant.

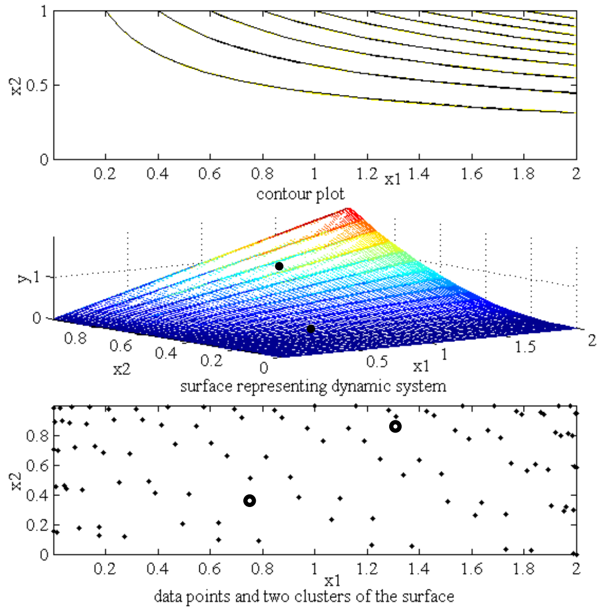

Fig. 16. A nonlinear surface of system dynamics subject to fuzzy modeling by special fuzzy membership functions [8].

IV. CONCLUSIONS

The precision and consequently accuracy of fuzzy modeling rapidly diminishes in multivariable modeling environment due to curse of dimensionality. In such cases, to improve the model performance several propositions are made in the literature such as similarity measures [9], evolutionary algorithms [10] or others [11]. Due to the model performance degradation, the majority of fuzzy modeling applications reported are in one-dimensional space and there is only few investigations reported on the degradation of fuzzy modeling in multidimensional case due to complexity involved and reconstruction error occurring as result of multivariable membership function projections. In contrast with this relatively weak attention, there is abundance of theoretical reported elaborations to be implemented in the real-life applications and subject to verifications. Therefore this work points out the implications of multidimensional fuzzy modeling as to its performance in the view of nonlinear systems. Fuzzy modeling is an essential machinery of soft computing which deals with machine intelligence via computational methods. Soft computing is especially effective in the applications in soft sciences. In particular, in the consideration of engineering or architectural designs, soft aspects generally pose ill-defined issues in highly multidimensional space due to nonlinearity and complexity. For soft issues model precision is tough to maintain and the multidimensionality is generally high. As to nonlinearity one of such soft areas is the architectural design [12]. The fuzzy logic approach implies essential contribution for the improvement of the design performance in the areas of design and therefore understanding fuzzy model efficiency on dealing with nonlinearity implies design enhancement which is a subject matter of importance motivating this research.

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