Sustainable Conceptual Building Design using a Cognitive System

BITTERMANN Michael S.
Delft University of Technology, The Netherlands
m.s.bittermann@tudelft.nl

Abstract. A cognitive system for conceptual building design is presented. It is based on an adaptive multi-objective evolutionary algorithm. The adaptive approach is novel and, in contrast with conventional multi-objective evolutionary algorithms, it explores the solution space effectively, while maintaining diversity among the solutions. The suitability of the approach for conceptual design of a multi-purpose building complex is demonstrated in an application. In the application, the goal of maximizing sustainability is treated by means of a model, which is established using neural computations. The approach is found to be suitable for addressing the soft nature of the sustainability concept. Also, the capability of the approach to distinguish among the alternative solutions from an unbiased viewpoint, i.e. without committing a-priori to a relative importance among the performance aspects, is demonstrated.

1. Introduction

In building design tasks generally several goals are to be satisfied at the same time. Some of the goals may be soft, that is they involve vagueness and imprecision. Examples of such goals are "high functionality" and "high sustainability". Due to their soft nature, establishing their relative importance in a design task is difficult to accomplish with certainty, in particular prior to having information on the inevitable trade-offs that are inherent to the task. This entails that cognition is required to decide about the ultimate suitability of solutions. Before elaborating this it is noted that soft goals also entail that the objectives may not be combined into a single criterion, i.e. the design problem is not to be treated as a single-objective optimization problem. Therefore, in general multi-objective optimization is an appealing approach for design purposes. To deal with

CAAD Futures 2011 : Designing Together, ULg, 2011
© P. Leclercq, A. Heylighen and G. Martin (eds)

the increasing complexity of the multi-objective optimization tasks *direct* search methods that are based on a population of solutions became the dominant approach for these tasks. The directness of the methods refers that objective function values are used to drive the search in contrast to the gradient-based methods, where derivative vectors are used for this purpose. The most prominent direct, population-based algorithms are evolutionary algorithms (MOEAs) and particle swarm optimization algorithms (PSO), and these are extensively being investigated for solving associated optimization problems, e.g. [1, 2, 3]. Population-based algorithms are particularly suitable for multi-objective optimization, since they evolve simultaneously a population of potential solutions.

Despite their effective use in many engineering areas the direct multi-objective optimization methods have drawbacks when applying them in the design area. In particular it should be clearly noted that they are able to handle only a moderate number of objectives to be simultaneously optimized. In general this number is not more than four or five. This is due to the nature of the Pareto ranking concept that is driving the multi-objective search process, and this will be explained below. Buildings host diverse human activities and thereby need to satisfy a myriad of elemental demands. Such demands range from desirable distances among related functions, to preferences for daylight, view, sustainability related aspects, and form related preferences. In order to let a direct search algorithm cope with such a large amount of requirements it should be noted that elemental requirements are constituents of higher-level requirements. For example the functionality of the individual building components constitutes the functionality of the building as a whole. Such aggregation of lower level aspects to form more abstract higher level aspects is a faculty of human reasoning. Apparently the human mind commonly uses such aggregations of attributes in order to cope with the amount of information. This is particularly necessary when aiming to take decisions that require cognition beyond mere intelligence. This occurs for instance when a number of alternatives should be compared regarding high-level, abstract goals like sustainability or functionality, which cannot easily be compared with each other.

Although cognition and intelligence are usually not distinguished in the literature due to their elusive nature, in this work the position is taken that cognition is a matter of addressing second order aspects, like the relative importance among the high-level goals, whereas intelligence deals with the discernment of solutions that satisfy such goals. With this understanding cognition requires intelligence as a prerequisite to pinpoint the range of basically suitable solution alternatives, whereas intelligence can occur without the cognitive process. As an example of the latter we can consider the conventional multi-objective optimization algorithms, where merely optimality for goals is pursued without addressing the relative importance among these goals at all in the process.

Artificial cognition is a novel concept that requires that the direct search methods are able to deal with goals having a high degree of abstraction. Next to this, for cognition to be effectively executed, it is necessary to have as basis a diverse set of solutions alternatives. Accomplishing this is not straightforward, since the strict search of non-dominated regions in the multi-objective solution space prematurely excludes some of the potential solutions resulting in aggregated solutions in this very space [4, 5]. This work addresses this issue by employing adaptive relaxation of the dominance concept during the search process. With respect to existing work employing relaxed Pareto ranking without adaptation, the adaptive relaxation is a novelty in particular aiming at tasks with many objectives. The effectiveness of the adaptive relaxation approach is demonstrated with an application from the domain of architectural design, where optimal spatial configurations satisfying multiple objectives are pursued during the conceptual phase. In the application, human-like reasoning is employed during the fitness evaluation. Based on this, artificial cognition is invoked, so that ultimate selection among Pareto solutions by a decision maker is facilitated. In particular the sustainability concept is treated in this work using computational intelligence, so that expert knowledge on sustainability is effectively integrated into a model. This is a novel approach to sustainability, since up till now the vague nature of the concept is not taken into account.

The paper is structured as follows. Section two of this paper describes the adaptive relaxation of the Pareto dominance concept. Section three describes the application of the adaptively relaxed search algorithm for sustainable conceptual building design. This is followed by conclusions.

2. Adaptive multi-objective optimization

In Multi-objective (MO) algorithms that are population-based, such as MOEA or Multi-objective Particle Swarm optimization (MOPSO), in many cases the use of Pareto ranking is a fundamental selection method. Pareto ranking aims to reach a solution surface in a multidimensional solution space, where the space is formed by multiple criteria representing the objectives. This surface is known as *Pareto front*. On this surface, the solutions are diverse but they are assumed to be equivalently valid. The driving mechanism of the Pareto-ranking based algorithms is the conflicting nature of criteria, i.e. increased satisfaction of one criterion implies loss with respect to satisfaction of another criterion. Therefore the $\boldsymbol{F_1}$, $\boldsymbol{F_2}, \ldots, \boldsymbol{F_N}$ which are of the form

$$F_i(\boldsymbol{x}) = f_i(\boldsymbol{x}) + \sum_{j=1,j\neq i}^{j=N} a_{ji} f_j(\boldsymbol{x}), i = 1,2,...,N \qquad (1)$$

where $\boldsymbol{F_I(X)}$ are the new objective functions; $\boldsymbol{A_{JI}}$ is the designated amount of gain in the $\boldsymbol{J}$-th objective function for a loss of one unit in the $\boldsymbol{I}$-th objective function. To find the Pareto front of a maximization problem we assume that a solution parameter vector $\boldsymbol{X_1}$ dominates another solution $\boldsymbol{X_2}$ if $\boldsymbol{F(X_1)\geq F(X_2)}$ for all objectives. At the same time a contingent equality is not valid for at least one objective.

In solving multi-objective optimization tasks, with the increase of the number of objective functions, i.e. with high dimensionality, the effectiveness of the Pareto ranking in this strict form is hampered. Namely, with many objectives, there are few solutions that dominate others in the strict sense expressed by (1). This means the search has little information to distinguish among solutions, so that the selection pressure pushing the population into the desirable region is too low. Therefore the algorithm prematurely eliminates potential solutions from the population, exhausting the exploratory potential of the population. As a result the search arrives at an inferior Pareto front with aggregation of solutions along this front. This issue can be understood considering that the conventional Pareto ranking implies a kind of *greedy* algorithm, which considers the solutions at the search area delimited by orthogonal axes of the multidimensional space, i.e. $\boldsymbol{A_{JI}}$ in (1) becomes zero. This is shown in Figure 1 by means of the orthogonal lines delimiting the dominated region. The point P in Figure 1 is ultimately subject to identification as an ideal solution. To increase the pressure pushing the Pareto surface towards to the maximally attainable solution point is the main problem, and adaptive relaxation of the orthogonality is an appealing solution and applied in this work. Although, some relaxation of the dominance is addressed in literature [6, 7], in a multidimensional space to identify the size of relaxation corresponding to a volume is not explicitly determined. In such a volume next to non-dominated solutions, dominated but potentially favorable solutions, as described above, lie. To determine this volume optimally as to the circumstantial conditions of the search process is a major and a challenging task.

In this work the solution for this task is due to the mathematical treatment of the problem, where the volume in question is identified adaptively during the search that it yields a measured pressure to the Pareto front towards the desired direction, at each generation as follows. The fitness of the solutions is ranked by the fitness function given by (2).

$$\boldsymbol{R_{fit}} = \frac{1}{N(\theta)+n} \quad , \qquad (2)$$

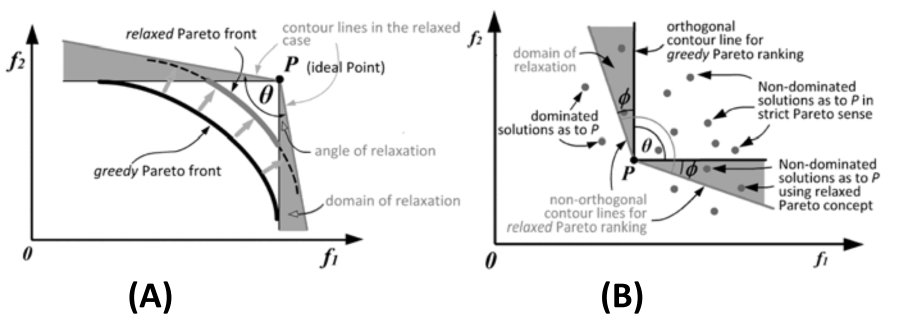


Fig. 1. Contour lines defining the dominated region in relaxed versus greedy Pareto dominance case (a); Implementation of the relaxation concept during the evolutionary search process (b).

where n is the number of potential solutions falling into the search domain consisting of the conventional orthogonal quadrant, with the added areas of relaxation. To obtain n in (2), for each solution point, say P in Figure 1, the point is temporarily considered to be a reference point as origin, and all the other solution points in the orthogonal coordinate system are converted to the non-orthogonal system coordinates. This is accomplished by means of the matrix operation given by (3)

$$F=\begin{bmatrix}F_1\\F_2\\\cdots\\F_n\end{bmatrix}=\begin{bmatrix}1\ a_{21}\ \ldots..\ a_{n1}\\a_{12}\ 1\ \ldots..\ a_{1n}\\\ldots\ldots\ldots\\a_{1n}\ a_{2n}\ \ldots..\ 1\end{bmatrix}\begin{bmatrix}f_1\\f_2\\\ldots\\f_n\end{bmatrix}=\begin{bmatrix}1 & \tan(\phi_2)\ \ldots.. & \tan(\phi_n)\\\tan(\varphi_2) & 1\ \ldots.. & \tan(\varphi_n)\\\ldots\ldots\ldots & & \\\tan(\theta_2)\ \tan(\theta_n) & \ldots.. & 1\end{bmatrix}\begin{bmatrix}f_1\\f_2\\\ldots\\f_n\end{bmatrix} \tag{3}$$

where the angles ϕ, φ, … θ represent the respective relaxation angles between one axis of the coordinate system and the other axes. After coordinate transformation using (3), all points which have positive coordinates in the non-orthogonal system correspond to potential solutions contributing to the next generation in the evolutionary computation. If any point possesses a negative component in the new coordinate system, the respective solution does not dominate P and therefore is not counted. This is because otherwise such a solution may lead the search in a direction away from P. The importance of this coordinate transformation becomes significant especially with greater amounts of objective dimensions.

Determination of the suitable relaxation angle depends on the particular conditions occurring during the stochastic search process. Adaptively changing the angle implies that the size of the relaxation domain used to grade an individual is considered in perspective with the relaxation domains presently associated to the other solutions in the population. This is implemented by means of (4), where the ratio between the relaxation angle and average relaxation angle is used. N(θ) in (4) can be considered as expressing the amount of virtual solutions that are accrued to the counted number of dominant solutions denoted by n in (2).

$$N(\theta) = \frac{s}{1 + \left(\theta / \bar{\theta}\right)} \tag{4}$$

This reflects the fact that when we take the greedy dominance concept, solutions that are dominated by s more solutions may turn out to be favorable in later stages of the search process, although they normally would already have been eliminated due to greediness of the conventional Pareto ranking scheme.

Considering (2) and (4) together it is clear that the purpose is to reward a chromosome for affording a wide relaxation angle θ, relative to the average angle of the population $\bar{\theta}$, and still having a low dominance count, denoted by n. The wide angle is to provide more diversity in the population for the next generation. However, when the relaxation angle would be excessively large, the population for the next generation can be crowded with trivial solutions. To prevent that, in (2) the number of non-dominated solutions with respect to the particular solution considered denoted by n, is summed up with the function of the angle N(θ). This means that between two solutions with the same amount of non-dominated solutions, the one with the wider angle is preferred. This is done for every solution in the population. This implies that the average angle $\bar{\theta}$ is changing for every generation adaptively. It is noted that the number s appearing in (4) is a constant number, used to adjust the relative significance of relaxation angle versus count n. This means the value of s should be selected bearing in mind particularly population size, so that solutions using wide angles are adequately rewarded, for instance.

3. Application for sustainable conceptual building design

3.1. Generating solutions

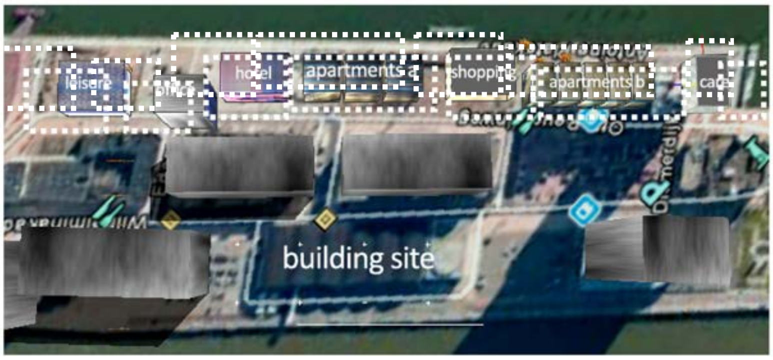


Fig. 2. Design objects subject to optimal positioning on the building site.

A layout problem of a building complex is considered, where the spatial arrangement of a number of spatial units is to be accomplished in such a way that three main goals are satisfied simultaneously. These goals are maximizing the building’s energy performance, functionality, as well as its performance regarding form related preferences. Taking the energy performance into account already during conceptual design is an important factor for enhanced sustainability of the building. The building subject to design consists of a number of spatial units, referred to as design objects, where every unit is designated to a particular purpose in the complex. The task is to locate the objects optimally on the construction site with respect to the three objectives forming suitable spatial arrangements. The objects are seen in Figure 2. It is noted that such layout problems are very generic forming an essential aspect of most design tasks in architecture, in particular during the conceptual phase. It is also noted that despite the low number of building elements in the present task, the amount of possible solutions is excessive, so that exhaustive search of the possibility space is not feasible.

In order to let the computer generate feasible solutions, it is necessary to ensure that spaces do not overlap, and objects should be adjacent to the other objects around and above it. This is realized in the present application by inserting the objects in a particular sequential manner into the site. This is illustrated in Figure 3. Starting from the same location, one by one the objects are moved forward, i.e.

translated in northern direction, until they reach an obstacle. An obstacle is either the site boundary or another object previously inserted. When an object touches a previously inserted one, the former object changes its movement direction from the northern to the eastern direction, moving east until again it reaches the site boundary or another object. As a final movement step the inserted object will move downward until it touches the ground plane. Packing objects in two dimensions in this way is known as *bottom-left two heuristic packing routine* in literature, e.g. [8]. It is noted that the objectives in conventional packing problems significantly differ from architectural design problems. For instance in the former case the adjacency or nearness of objects is usually irrelevant, whereas in building design it is relevant.

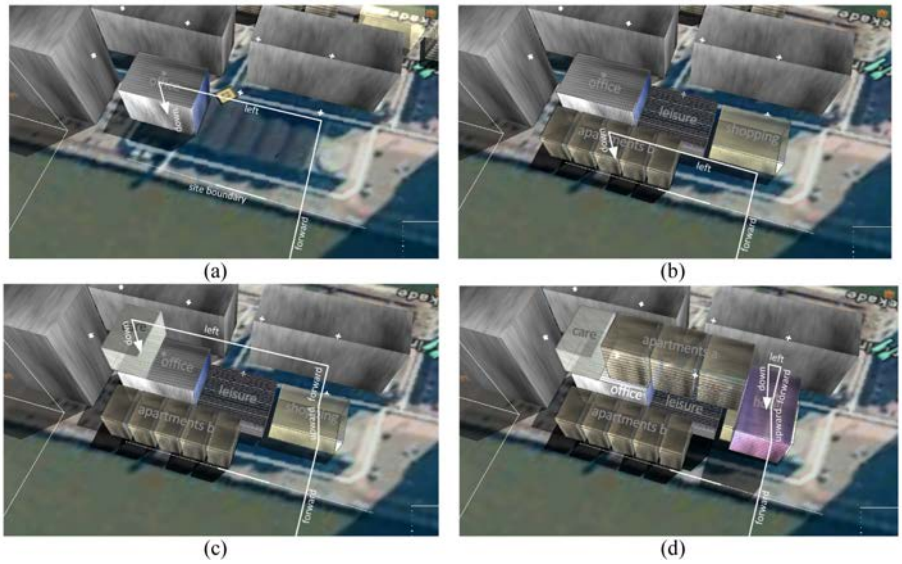


Fig. 3. Generation process of a solution through sequential insertion.

3.2. Evaluating the performance of solutions

In order to deal with the abstract nature of goals involved in the task, a computational intelligence approach is used to assess the suitability of a solution through simulating a human-like reasoning process. The approach is based on a neural structure having intelligent logic processors embedded into it, so that the system performs as a Takagi Sugeno (TS) type fuzzy model [9]. The model structure is a *neural tree* structure. It is composed of terminal nodes, non-terminal nodes, and weights of connection links between two nodes. The non-terminal nodes represent an artificial neuron, and the neuron type is an attribute introducing a non-linearity simulating a neuronal brain activity. In this approach, the non-linearity is established by means of a Gaussian function, which has several desirable features for the intended goals; namely, it is a radial basis function ensuring a solution as well as the smoothness. At the same time it plays the role of a fuzzy membership function, so that the system is to be considered a fuzzy logic system, as its outcome is based on fuzzy logic operations and thereby associated reasoning [10]. It is clearly noted that the fuzzy neural tree used here is not a decision tree : a node of the fuzzy neural tree executes a logic operation simulating a neural activity, whereas a node in a decision tree represents a choice or a consequence among alternative decisions. It is further noted that the neural tree approach is different compared to outranking approaches, such as PROMETHEE [11] : in the latter case pair-wise comparisons forms the foundation of the evaluation, and specification of a preference functions is required by the decision maker. The neural tree model does not involve preference functions, but it uses the concept of membership function from fuzzy logic, and these functions are optimized based on a consistency condition that is inherent to the knowledge subject to modeling.

The root node of the neural tree shown in Figure 4 describes the ultimate goal subject to maximization, namely the design performance and the tree branches form the objectives constituting this goal. The connections among the nodes have a weight associated with them, as seen from the figure. The weights are given by a decision maker, specifying the relative significance a node has for the node one level closer to the root node as an expression of knowledge. The weights and input information is processed at each inner node of the tree model based on a fuzzy logic operation given by (5), where O_j is the output of the j-th inner node, μ_i denotes the strength of the i-th input coming to this node; w_{ij} denotes the relative importance of this input; and σ_j a parameter responsible to ensure a consistency in the AND operation, which is to be identified using classical or evolutionary optimization [12].

(5)

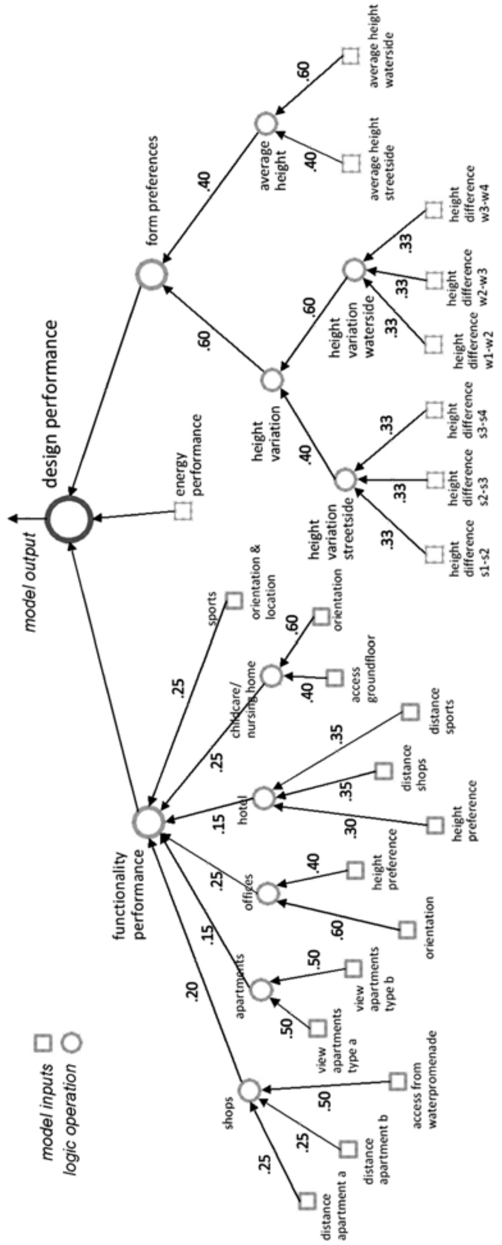


Fig. 4. Fuzzy neural tree for performance evaluation of the candidate solutions.

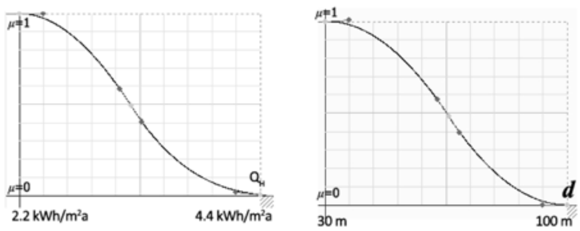


Fig. 5. Fuzzy memberhip functions at two of the terminal nodes in Fig. 4 for energy performance evaluation (a); for evaluation of the average building height along the waterfront (b).

In order to provide the neural model with input values, fuzzification processes are carried out at the terminal nodes shown by means of square shaped boxes in Figure 4. The fuzzification yields the degree of satisfaction for the elemental requirements in the form of membership degrees. Two examples of this operation are shown in Figure 5. For instance concerning the requirements on average height of the building the decision maker prefers to have a low average height along the street side to give the building a less dominant expression when perceived by people walking along the waterfront. This requirement is seen from the membership function in Figure 5b, where the membership degree diminishes with increasing height. The fuzzified information is then processed by the inner nodes of the tree. These nodes perform AND operations using multi-dimensional Gaussian membership functions as described above. The width-vector of a Gaussian reflects the relative importance among the inputs to the corresponding node. Executing the sequence of logic operations in parallel manner starting from the model inputs, the design performance is obtained at the penultimate node outputs of the model. This means the more satisfied the elemental requirements at the terminal level are, the higher the outputs will be at the nodes above, finally increasing the design performance at the root node of the tree.

3.3. Artificial cognition : distinguishing among Pareto optimal solutions

The neural model is used during the evolutionary search process to evaluate the fitness of solutions, in order to arrive at designs with maximal design performance. In the multi-objective implementation the outputs of the nodes *functionality*, *energy*, and *form preferences*, which are the penultimate nodes, are subject to maximization. Their values are used in the fitness determination procedure of an adaptive multi-objective genetic algorithm.

In accordance with the multi-objective treatment, from Figure 4, we note that the weights connecting the root node with the nodes one level below are not determined by a decision maker. We term these weights as w_1, w_2, and w_3. As the neural model can be considered as a Takagi Sugeno (TS) type fuzzy model [9] the computation of the performance score at the root node is a defuzzification in TS sense, given by

$$w_1 f_1 + w_2 f_2 + w_3 f_3 = p \,, \tag{6}$$

where f_1 is the output of the node *functionality performance;* f_2 of node *energy performance*; f_3 of node *form preferences.* The variable p denotes the design performance which is also requested to be maximized, which is the ultimate goal of the design. In much real-world decision making tasks the cognitive viewpoint plays an important role. This means it is initially uncertain what values $w_1,\ldots w_3$ should have. The node outputs $f_{1,\ \ldots,}\ f_3$ can be considered as the *design feature vector*, and the reflection of these features can be best performed if the weights w_1 ; …; w_3 define the same direction as that of the feature vector. This implies that the performance p_{max} for each genetic solution is given by [13].

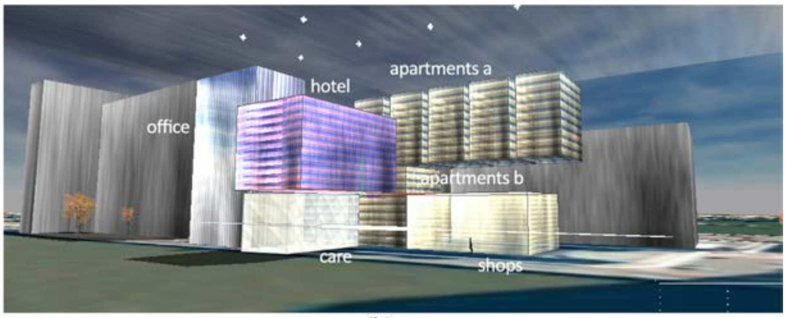
$$p_{\max} = \frac{f_1^2 + f_2^2 + f_3^2}{f_1 + f_2 + f_3} \,, \tag{7}$$

Therefore, p_{max} in (7) is computed for all the design solutions on the Pareto front. Then the *solution with maximal p_{max} performance* is selected among the Pareto solutions. This way the particular design is identified as a solution candidate with the corresponding w_1, w_2, …., w_n weights. These weights form a priority vector ***w****. One should note that, although the priority vector information is not used during the genetic optimization, a resulting Pareto front offers a number of design options with fair performance leaving the final decision dependent on second-order preferences. Using (7), these second-order preferences are identified that are most promising for the task at hand, in the sense that maximal solution performance is accomplished for any possible weight combination.

Distinguishing systematically among the Pareto optimal solutions in this way is to be considered as an act of machine cognition. This is because second order information is systematically obtained without contribution by the decision maker in this act, which is possible due to the particular mathematical nature of the TS defuzzification process. It is noted that the machine cognition does make use of decision makers' knowledge modeled through the neural computations that yield f_1,..f_3; however, as the Pareto optimal front is not known a priori, the higher level priority information, i.e. $w_1\ldots w_3$, is also unknown in advance, and subject to computational identification using Eq. (7), once the Pareto optimal front is established.

3.4. Application Results

(a)



(b)

(c)

Fig. 6. Pareto optimal designs, D1 (a); D2 (b); and D3 (c).

The adaptive evolutionary algorithm is executed and the Pareto front is obtained. In the execution *s*=16 in (4) is taken. To exemplify the solutions on the Pareto front, three resulting Pareto-optimal designs are shown in Figure 6. Fig. 6a shows design *D1*, 6b shows *D2*, and 13c shows *D3*. The design performance for the designs shown in Fig. 6 is shown in figure 7.

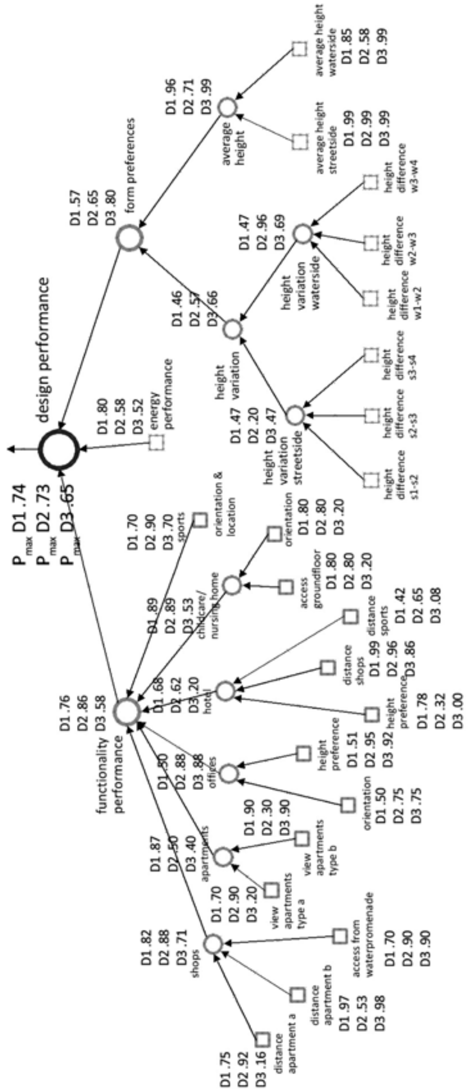


Fig. 7. Node outputs for the three Pareto optimal designs shown in Fig. 6.

From Figure 7 it is seen that D1 and D2 are quite close as to their p_{max} value, whereas D3 has significantly lower p_{max} design performance. This is despite the fact that D3 outperforms the other two designs regarding form performance. Both determinants of form performance, namely height variation and average height both score significantly higher for D3 compared to D1 and D2. Here particularly the height variation is decisive, since its relative importance is .60 vs. .40 for average height seen from Figure 4. For D3 both the height variation along the street side and along the waterside are relatively large (.47 and .69), as it is required. The reason why D3 is outperformed by the other two designs is because its functionality and energy performance are both relatively low. The low functionality performance of D3 is especially due to low functionality of the hotel (.20). This is also seen from Figure 6c, where it is noted that the hotel component has a low height, which is undesirable because the view from the upper hotel rooms is not that attractive. Also the distance between the sporting center and the hotel is undesirably close, which is expected to cause unwanted disturbance of the hotel guests. The inferior energy performance of D2 and D3 is because the perimeter of hotel and office is largely exposed, while in D1 hotel and office are more surrounded by other components. This causes more transmission heat loss in case of D1 and D2 compared to D3. D2 has the highest functionality performance of the three designs (.86), which is particularly due to high functionality for the offices.

From the results we also note that design *D1* outperforms the other two designs as to its p_{max} value, despite the fact that the other two designs perform better on the form aspect. This is explained from the fact that the Pareto front in the present decision making task is not symmetrical to the diagonal line in objective space, spanning from the origin to the ideal point at (1, 1, 1). The front is oriented in such a way that the form performance is generally lower compared to energy and functionality performance. This means that in the present task, in absence of a-priori second-order preferences, one should not consider form as important as energy and functionality. Considering that D1 and D2 are quite similar as to their maximal performance it is noted that solutions with either outstanding functionality performance, or solutions that perform well on energy and functionality at the same time, are worth to consider in the ultimate decision making. However it is noted that many Pareto optimal solutions are not significant from an unbiased viewpoint, that is their p_{max} value is inferior, e.g. in the case of D3. This information is uniquely obtained through the cognitive approach. This way the amount of solutions to consider in the selection among the Pareto solutions is reduced, and thereby the decision making task is alleviated.

4. Conclusions

An adaptive relaxation approach for enhanced multi-objective optimization is presented and demonstrated in a conceptual building design task, where sustainability concerns are taken into account. In this approach the Pareto dominance concept is adaptively relaxed during an evolutionary search process. The adaptation is found to yield diverse Pareto optimal solutions. In the application a building consisting of several volumes is obtained, so that three soft objectives pertaining to the building are satisfied. The satisfaction degree for each goal is measured using a neural model, so that the adaptive evolutionary algorithm makes use of some human-like reasoning during its search. Since the evaluation model is a Takagi Sugeno type Fuzzy model, the defuzzification process at the model output for the multiple objectives yields new information regarding the relative suitability of solutions from an unbiased viewpoint. For example, from the application it is seen that *form* related performance plays a less significant role compared to *energy* and *functionality* concerns. This information is relevant in the selection among the Pareto optimal solutions, and it is challenging to be obtained otherwise. Provision of this information constitutes an act of machine cognition, because the seemingly equivalent optimal solutions are systematically distinguished by means of the computations without involvement of the decision maker. Computational cognition may be significant, as it enhances the decision making in soft problems, so that ultimate choices can be made with great awareness and confidence despite the softness.

References

1. Zitzler, E. & Thiele, L. (1999). Multiobjective evolutionary algorithms : A comparative case study and the strength Pareto approach. IEEE Trans. Evolutionary Computation 3(4) : 257-271.
2. Engelbrecht, A.P. (2005). Computational Swarm Intelligence. Chichester, GB, Wiley.
3. Deb, K. (2001). Multiobjective Optimization using Evolutionary Algorithms, John Wiley & Sons.
4. Sato, H., Aguirre, H.E. & Tanaka, K. (2007). Controlling Dominance Area of Solutions and Its Impact on the Performance of MOEAs. 5-20.
5. Laumanns, M., Thiele, L., Deb, K., et al. (2002). Combining convergence and diversity in evolutionary multiobjective optimization Evolutionary Computation 10(3) : 262-282.
6. Branke, J., Kaussler, T. & Schmeck, H. (2000). Guiding multi-objective evolutionary algorithms towards interesting regions. Technical Report TR No. 399, AIFB University of Karlsruhe, Germany.
7. Deb, K., Sundar, J., Bhaskara, U, et al. (2006). Reference point based multi-objective optimization using evolutionary algorithm. Int. J. Comp. Intelligence Research 2(3) : 273-286.
8. Hopper, E. & Turton, B.C.H. (2001). An empirical investigation of metaheuristic and heuristic algorithms for a 2D packing problem. Eur. J. Oper. Res. 128 : 34-57.
9. Takagi, T. & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics 15 : 116-132.
10. Hunt, K.J., Haas, R. & Murray-Smith, R. (1996). Extending the functional equivalence of radial basis function networks and fuzzy inference systems. IEEE Trans. Neural Networks 7(3).

11. Brans, J.-P. & Vincke, P. (1985). A preference ranking organisation method : The PROMETHEE method for MCDM. Management Science 31(6) : 647-656.
12. Ciftcioglu, Ö., Bittermann, M.S. & Sariyildiz, I.S. (2007). A neural fuzzy system for soft computing. NAFIPS 2007, San Diego, USA, IEEE.
13. Bittermann, M.S. & Ciftcioglu, O. (2009). A cognitive system based on fuzzy information processing and multi-objective evolutionary algorithm. IEEE Conference on Evolutionary Computation - CEC 2009. Trondheim, Norway, IEEE.