Blur in Human Vision and Increased Visual Realism in Virtual Environments

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Abstract. A challenge for virtual reality (VR) applications is to increase the realism of an observer's visual experience. For this purpose the variation of the blur that an observer experiences in his/her vision, while he/she focuses on a particular location, can be mimicked by blurring the VR computer graphics based on a model of the blur. The blur in human vision is the result of a combination of optical and neural vision processes; namely optical refraction, non-uniform retinal sampling, and cortical magnification. Due to the complexity of the phenomenon, apparently no theoretical model of the blur has been published. In this work we model the combined effect by means of a probabilistic model of the human visual system. The results from the models match common vision experience verifying the validity of the underlying theoretical considerations. The implementation of the model for increased realism in virtual reality is illustrated by means of a rendering of a virtual reality scene, which is processed for two different acts of focusing.

1 Introduction

Traditional applications of virtual reality (VR) are medical surgery training, flight and driving simulation, as well as industrial and architectural design [1, 2]. In the applications usually a high degree of visual realism is desired for increased effectiveness. Visual realism concerns both, the experienced egocentric and exocentric distances, and the level of visual detail of the environment. A source reducing visual realism of VR experience is the fact that a virtual scene is usually rendered on a 2-dimensional display, so that an observer has some awareness of the display surface [3]. There is evidence that awareness of the display surface interferes with the perceived three-dimensionality of a scene displayed [4, 5]. This evidence appears to corroborate with experiments where absolute egocentric distances are found to be generally underestimated in VR [6, 7], while a number of possible reasons for this can be ruled out [8, 9], including the quality of the computer graphics as claimed by [10].

The visual experience of an object differs from the experience of the object's counterpart in VR. One difference is the experienced variation of visual detail on the object when we focus our vision to a certain location on it. This phenomenon we refer to as *blur* phenomenon. When we focus on a surface patch S_1 of an object, we experience the patch as visually *sharp*. When we keep focusing on S_1, we experience another patch S_2 located at some distance from S_1 as less sharp, i.e. *blur*. The degree of experienced blur depends on the geometric relation among S_1, S_2, and the location of

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observation. We can verify this by increasing the distance between S_2 and S_1, while we continue to focus on S_1. When we experience an environment in virtual reality we obtain the visual information looking at the surface of the display. While looking at a patch of the display surface the degree of blur we experience for another patch on the surface is clearly dependent on the geometry of the display surface, and not on the geometry of the virtual objects being displayed. Therefore it is clear that the variation of blur we experience viewing the VR rendering may differ significantly from what we would experience if the virtual scene were physically present. This mismatch may be part of the reason why comprehension of the geometry of virtual environments is usually not accurate.

In this work we endeavor to establish a model of the blur occurring in human visual experience of an environment. Based on this model renderings of virtual environments can be processed in such a way that visual experience of them matches more closely to the experience of the corresponding physical environment. The blur phenomenon in human vision is a combined effect of several processes that occur in the human vision system. The processes include refraction of light by cornea, lens, and vitreous, non-uniform retinal sampling, and cortical magnification. The optical refraction of the light yields a *gradient of focus* of the image reaching the retina. The dependence of the focal gradient on distance is well known [11]. The model of focal gradient involves a Gaussian point-spread function characterized by the standard deviation σ. It models the relation between the distance of a location in the environment and the degree of defocus the corresponding image has on the retina, given the lens properties of human eye. The image with its gradient of focus is then processed by the photoreceptors on the retina. The retina has a non-uniform distribution of photoreceptors, i.e. the amount of cone and ganglion cell density is reduced in the periphery of the retina [12] with respect to the central region termed fovea. The non-uniformly sampled retinal image is thereafter mapped to the striate cortex, where another non-linearity is introduced into the processing of the stimulus known as cortical magnification. Cortical magnification refers to the fact that with eccentricity from the center location of the retina, less and less portion of the striate cortex processes the corresponding retinal information [13, 14]. Non-uniform retinal sampling and cortical magnification are considered to cause the well-known sharp decrease of acuity with eccentricity [15], and it has been found that the decrease is proportional to the cortical magnification factor. Since the blur a human experiences in his/her vision is a combined effect of optical and neural processes, modeling the variation of blur is a challenging issue. This is because it is uncertain how the optical effects and neural effects interact precisely, so that the dependence of the blur experience on the geometry of the environment remains essentially unknown.

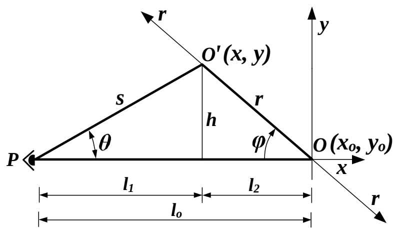
Due to the uncertainty mentioned above in this work we do not attempt to model each component of the visual system and thereafter combine them to develop a model of blur. In our approach we regard the visual system as a complex system, where the input is the visible environment and the output is the mental realization of the environment. In the approach we distinguish two types of human vision experience that are related. The first type is perception of an environment, where no particular item is intentionally looked at, yet an initial awareness of the visible environment is built up. We refer to this type of experience as perception during *early vision*. The second type of experience is a specific conditioning of the perception during early vision. The condition is that a certain item is gazed at, while the observer has attention for the other objects in view. Given that a certain location in view is gazed at, the attention for the other items in the visual scope takes a certain form. In this *conditional perception* the resulting human vision experience takes a different form compared to the early vision case, namely it yields the experience of blur in human vision. Since the model of the conditional perception is based on the model of perception during early vision, we start by modeling perception during early vision. This is described in section 2. The conditional perception and resulting blur is modeled in section 3. Thereafter an application of the model of blur is presented in section 4. This is followed by conclusions.

2 A Model of Perception During Early Vision

Our subjective impression is that we view our environment in sharp clear focus. In particular, when we assimilate an overall comprehension of a scene we are usually not aware of the variation of visual sharpness that accompanies an intentional act of looking someplace. The phase of initial formation of environmental awareness is known as early vision [16, 17]. Early vision involves eye saccades, optical, retinal and neural processes etc. while an observer is apparently not aware of each act of looking he/she exercises during this phase. This unawareness must be due to complex processing of retinal stimuli in the brain, where attention is selectively paid to environmental information in such a way that an overall awareness of the visible environment is assembled. This means in early vision an observer is viewing locations in the environment in such a way that the net result is equivalent to the event that the observer viewed the entire environment in his/her visual scope. Without prior information about the scene an observer is about to view, he/she pays attention to any direction in his/her visual scope equally. Please note that equally paying attention does not refer to uniform sampling of light stimulus or uniform cortical mapping. It refers the combined effect of the processes interacting in the visual system during early vision. The combined effect results in the phenomenon, that no differential angle in our visual scope has a greater chance to yield awareness of an item that is located within this angle during early vision. In this sense all angles in our view are equivalent at the first instance. Based on this consideration a probabilistic model of early vision is developed, where the *visual attention* paid with respect to vision angle is modeled by means of a uniform probability density function (pdf) [18]. This mathematical definition of attention substantiates the existing verbal definitions of the concept. Any point of a visible environment "receives" a particular degree of the attention paid. To compute the distribution of the observer's attention over the environment we consider a fundamental visual geometry shown in figure 1. In the figure an observer at the point P is viewing an infinite plane whose intersection with the plane of page is the line passing from two points designated as O and O'. O represents the origin. We compute the variation of attention along the r axis shown in figure 1a. The pdf $f_r(r)$ of this random variable is computed using the theorem on the function of random variable. The resulting pdf is [18]

$$f_r(r) = \frac{l_o}{\pi/2} \frac{\sin(\varphi)}{r^2 - 2l_o r\cos\varphi + l_o^{\ 2}} \quad \text{for} \quad 0 < r < \frac{l_0}{\cos(\varphi)}. \tag{1}$$

Equation 1 gives the variation of visual attention during early vision along the axis *r*. To compare this variation for different angles of φ we obtain the variation of attention with respect to the direction parallel to the *y*-axis. Figure 1b is the magnified portion of figure 1a in the vicinity of the origin indicating the relationship between the infinitesimally small distances *dy* and *dr*.



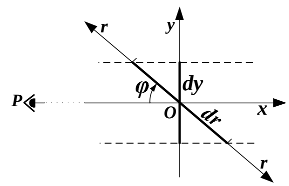


Fig. 1. The geometry of visual perception, where the observer has the position at point *P* with the orientation to the point *O*. The *x,y* coordinate system has the origin placed at *O* (a); the magnified region at the origin (b).

The relation between *dy* and *dr* is given by $dy = dr\ sin(\varphi)$ or

$$dr = dy/sin(\varphi) \tag{2}$$

Substitution of (2) into (1) yields

$$f_r^*(r) = \frac{l_o}{\pi/2} \frac{1}{r^2 - 2l_o r\cos\varphi + l_o^{\ 2}} \tag{3}$$

The plot of $f_r^*(r)$ in Cartesian coordinates is shown in figure 2a. Based on the probabilistic expression of attention, perception is defined as the integral of attention over a certain geometric domain [18].

3 A Model of the Blur in Human Vision

Based on the result from the previous section we aim to model the situation when an observer gazes at an object. In this case there is still perception of other objects within the visual scope. Considering the probability density $f_r(r)$ given by (3), we can derive a conditional density conditioned by the event of seeing point *O* in figure 1. This means the observer is gazing at point *O* while he still has some attention for *O'*. This conditional attention can be calculated by means of Bayes's theorem yielding the probability density along the line *OO'* while the observer focuses at the point *O*. We note that this application of the Bayes' theorem differs significantly from the existing Bayesian approaches to perception. In contrast to the *probabilistic model* we develop in this paper, the Bayesian approach to perception in literature is a statistical inference approach carried out in terms of *probability* statements. The vast majority of probabilistic approaches in the literature for perception modeling is based on the Bayes' rule. The present probabilistic approach is a complementary endeavor for the Bayesian approach providing theoretical base for the prior density. Therefore the posterior density can be obtained more accurately in the Bayesian approach.

The computation of visual blur is accomplished as follows. The probability density $f_r^*(r)$ in (3) is also a function of the angle φ. Therefore we can consider that it is a function of two variables, which are r and φ, and thereby we write it in the form $f_r^*(r, \varphi)$, which is given by

$$f_r^*(r,\varphi) = \frac{l_o}{r^2 - 2l_o r\cos\varphi + l_o^{\,2}} \;. \tag{4}$$

Above the factor $2/\pi$ is omitted taking the angle θ as a unit angle, so that $f_\theta=1$ as compared to $f_\theta=1/(\pi/2)$ [18]. One can interpret that f_r is a marginal joint probability density of a two dimensional joint probability density. This means $f_r(r|\varphi)$ gives the probability density with respect to r at the angle φ. With this understanding, there is no objection to apply the Bayes' theorem for the joint probability density. Accordingly we write

$$f_r(r\,|\,\varphi) = \frac{f_r(r,\varphi)}{f(\varphi)} \tag{5}$$

where $f_r(r|\varphi)$ is the conditional probability density with respect to r; $f(\varphi)$ is the marginal probability of the joint probability density $f_r(r,\varphi)$ at the angle φ, and therefore the Bayes' theorem is applicable without any difficulty. It is noteworthy to mention that in the continuous case the formulation of conditional probability presents some difficulty since the theorem requires a probability at the denominator rather than a probability density, which is the case in the continuous case. This is described in [19]. From (5), we obtain $f(\varphi)$ as [18]

$$f(\varphi) = \left[\arctan\frac{\tan(\theta)\tan(\varphi)-1}{\tan(\theta)+\tan(\varphi)}\right] + \frac{\pi}{2} - \varphi \tag{6}$$

where $tan(\theta)$ is

$$\tan(\theta) = \frac{r\sin(\varphi)}{l_o - r\cos(\varphi)} \tag{7}$$

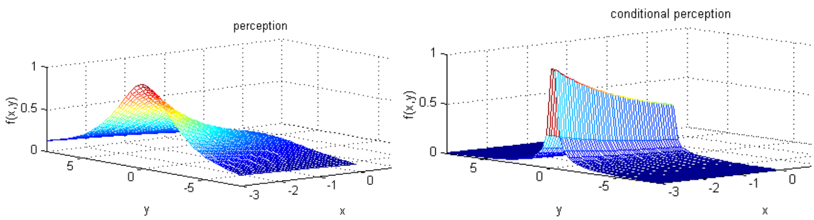
Substitution of (6) and (2) into (4) yields

$$f_r^*(r\,|\,\varphi) = \frac{l_o}{f(\varphi)}\,\frac{1}{r^2 - 2l_o r\cos\varphi + l_o^{\,2}} \,. \tag{8}$$

The probability density $f_r^*(r|\varphi)$ gives the conditional attention, i.e. the attention conditioned on φ, while the observer focuses on the point *O*. This attention we define as sharpness. The inverse of sharpness we define as *blur.*

The visual attention $f_r^*(r,\varphi)$, that is perception per unit length, is given by (4) and presented in figure 2a. The variation of conditional perception per unit length $f_r^*(r|\varphi)$

given by (8) is presented in figure 2b. The latter probability density indicates the degree of attention for an object in the case where another object receives full attention. In other words, when we gaze at an object at *O*, which is located at the point $y=x=0$ in figure 2b, we still have attention for other objects with a lesser degree, so that becoming aware of a second object located at *O'* is subject to probability. This probability quantifies the degree of seeing the object at *O'*.



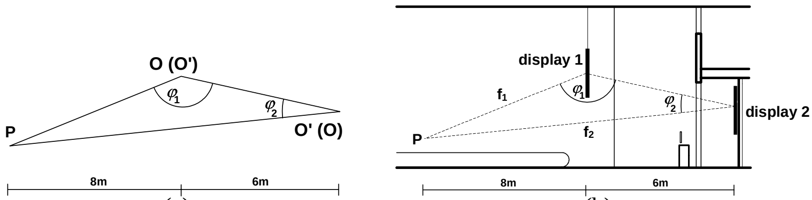
(a) (b)

Fig. 2. Computed perception per unit length (a) and conditional perception per unit length (b)

Comparing figure 2a and 2b we note that in the early vision case the attention, which is the perception per unit length, is distributed more evenly throughout space compared to the conditional perception case. This means the awareness for objects in the vision periphery diminishes gradually in early vision compared to the case, where an item is gazed at, so that awareness diminishes sharply.

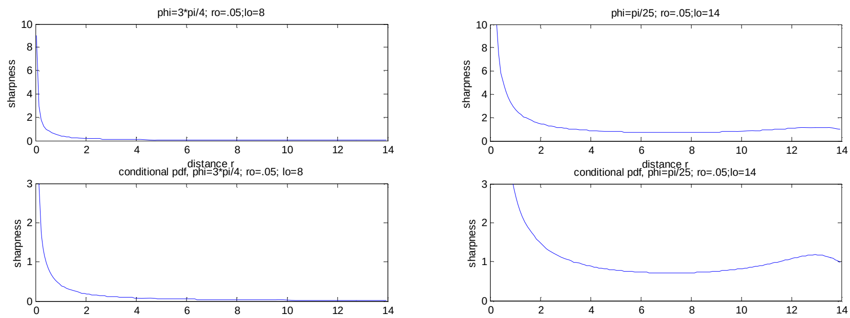
In the following we consider some implications of the pdf of conditional perception shown in figure 2b. As examples we consider two acts of focusing. These are shown in figure 3a. In the first case the focus is on point *O* shown in the figure and the conditional perception is considered along the line *OO'*. In the second case the focus is switched to the object *O'*, so that it becomes the new focus point labelled *(O)* in figure 3a, and the conditional perception is considered along the line between (*O)* and the other point labeled *(O')*. In the first case the angle $\varphi_1=3\pi/4$ and $l_o=8m$. In the second case $\varphi_2=\pi/25$ and $l_o=14m$. Figure 3b shows a vertical section of a scene subject to conditional perception, having the same geometry as shown in figure 3a.

The variation of sharpness with increasing distance *r* is obtained as the intersection of a vertical plane with the conditional perception surface presented in figure 2b, where the plane is oriented with φ and passing from the object *O*. Two plots of sharpness with respect to *r* for the two acts of focusing shown in figure 3a are shown in figure 4a and 4b. Figure 4a shows the sharpness along *r for* $\varphi=3\pi/4$, l_o=8m. Figure 4b shows the sharpness along *r* for $\varphi=\pi/25$, l_o=14m. In both cases the sharpness diminishes with the distance *r*, however in the first case it diminishes sharper than in the second case. In figure 4b we observe that for distances *r*>3m sharpness does not diminish as drastically as in figure 4a, but it remains practically constant at a certain level and even it increases as the distance increases towards $r=l_o$. This means, when we gaze at a nearby object, a second object located at a greater distance appears much more blur than in the reverse situation if we focus on the distant object. In the latter case the nearer object appears still relatively sharp. This is demonstrated in the application section.



(a) (b)

Fig. 3. (a) Sketch showing two acts of focusing: for $\varphi_1=3\pi/4,\ l_o=8m$ and $\varphi_2=\pi/25,\ l_o=14m$; (b) Vertical section of a scene subject to conditional perception, with the same geometry as in 3a



(a) (b)

Fig. 4. (a) Plot of sharpness with respect to the distance *r* for $\varphi=3\pi/4;\ l_o$=8m, where *r* corresponds to the line *OO'* in figure 3a; r_o is the distance at which the plot starts; (b) Plot of sharpness with respect to *r* for $\varphi=\pi/25;\ l_o=14m$, where *r* corresponds to the line (*O)(O')* in figure 3a

4 Application

The model of perception during early vision presented in section 2 has found application in robot navigation [20] and design [21] as reported earlier. In the following we investigate the application of the model of visual blur/sharpness presented in section 3 for VR environments with the objective to increase the visual realism of an observer's experience. The issue we are concerned about in the application is the influence blur has on depth perception. Virtual reality graphics are usually shown on a two dimensional display. An observer is looking at a particular pixel on the computer display at a given moment. This location can be obtained by means of eye tracking for example. Having this information, the degree of blur can be computed for every surface patch of a virtual scene, so that the rendered blur matches the stimulus an observer would obtain in case the scene the observer is experiencing were physically present. For each new act of looking the blur should be updated in real-time. To simulate the implementation we apply the results from the model of blur to a rendering of a virtual reality scene. The scene is the one shown in figure 3b, which belongs to the computations presented in figure 4. As shown in figure 3b we consider two objects of a scene as the targets of an observer's visual focus; the objects in our case are two flight schedule displays. These objects we take as an example for the blur phenomenon modelled. The size of the displays in the scene is equal. *Display 1* is located nearby to the observer, who is located at the place denoted by *P* as shown in figure 3b. *Display 2* is located beyond the first one. We compare two perception events of the observer: the first one is gazing at a place on display 1, and the second one gazing at a place on display 2. The gaze line belonging to the first event is shown in figure 3b by means of a dashed line labeled f_1. The gaze line belonging to the second event is shown by means of a dashed line labeled f_2. Figure 5 shows the rendering of the scene shown in figure 3b without blur. The rendering is then processed based on the result from the model presented in figure 4a and 4b and the result is shown in figures 6 and 7.

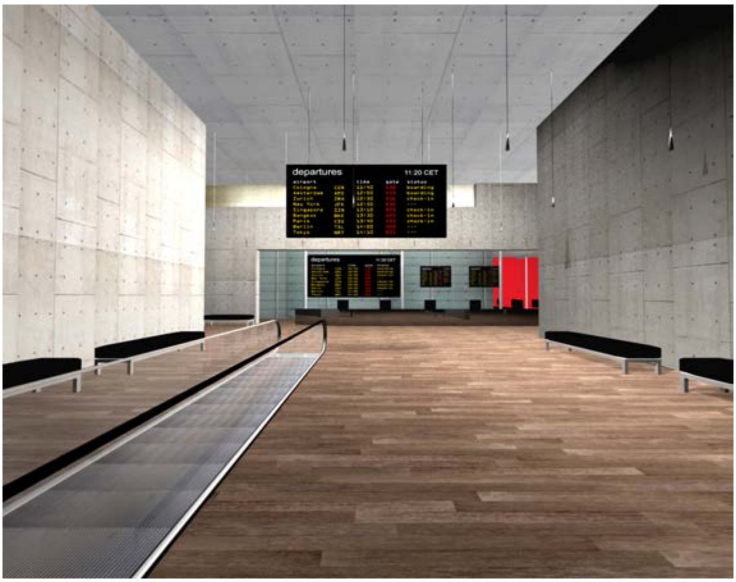


Fig. 5. Endering of the VR scene shown in figure 3b without perceptual blur

Figure 6 shows the event that the observer focuses on the plus sign on *display 1*. Figure 7 shows the event that the observer focuses on the plus sign on *display 2*.

Let us consider figure 6: When an observer is focusing on the plus sign on *display 1*, *display 2* is perceived with significant blur. This is because the angle φ_1 is quite large as shown in figure 3b, so that the sharpness quickly decreases with increasing distance, as shown in figure 4a. We can verify the increased realism by keeping our visual focus on the plus sign in figure 6 and at the same time perceiving *display 2*. Since the blur of *display 2* is excessive, we clearly get the impression that *display 1* is located nearer to us than *display 2*. Let us consider figure 7: When an observer is focusing on *display 2*, *display 1* is experienced as relatively sharp compared to *display 2* in the case shown in figure 6. This is because *display 1* is located nearer to the observer than *display 2*. The angle φ_2 shown in figure 3b is small, so that the sharpness of *display 1* is relatively high, as this is shown in figure 4b. This can be verified by keeping the visual focus on the plus sign in the figure and at the same time perceiving display 1. Again, one clearly has the impression that display 1 is nearer to our

Fig. 6. Rendering of the VR scene shown in figure 3b blurred based on the result presented in figure 4a; the blur is according to the event of focusing on the plus sign on display 1

Fig. 7. Rendering of the VR scene shown in figure 3b blurred based on the result presented in figure 4b; the blur is according to the event of focusing on the plus sign on display 2

viewpoint than *display 2*. Let us consider the blur of the floor and of the ceiling in figures 6 and 7. We note that locations directly below or above the focus point are relatively sharp compared to locations at the side regions of the floor or ceiling. This is because the angle φ for the locations directly above/below is quite small compared to the places on the side. For illustrative purposes this effect is exaggerated in the figures. It is noteworthy to mention that when the reader focuses on the plus signs in figures 6 and 7 an additional blur component occurs that should be taken into consideration in the preparation of a perception blurred image. Namely, when we focus on the page of the paper on which the scene is displayed, naturally a blur is induced that depends on the geometry of the paper surface. This additional blur should be accounted for by means of convolution of this blur with the blur that occurs due to the geometry of the scene. In order to avoid interference of this additional blur with the blur induced by the geometry of the virtual scene, the images shown in the paper should be observed from a large distance.

The added value of the perception based image processing is that the observer gains a better understanding of the geometry of the virtual scene compared to the case he/she is viewing a conventional rendering without perceptual blur. This is because the stimulus presented via a perceptually blurred image matches more closely to the perception in a corresponding physical environment than the matching to gazing at an image that is equally sharp everywhere. In order to experience the improved realism in figures 6 and 7 we emphasize that the reader keeps focusing on the plus sign shown in the figures. Certainly blur is only one of the depth cues relevant for comprehension of environmental geometry. However we have reason to consider the variation of blur a significant source of visual information on an environment. Every act of looking to a certain location yields a particular degree of blur for every patch of the visible surface of the environment. As the variation of blur depends on the geometry of the environment, every act of focusing yields a unique blur pattern that encodes the geometric constitution of the environment. Since there are a great many visible environmental patches within our visual scope there is a significant amount of information available for the brain to attain comprehension of an environmental geometry with every act of looking. With successive saccades, where different locations in the environment are focused, the amount of information increases, so that the environmental geometry is usually comprehended unambiguously.

5 Conclusion

A model of blur human experiences in his/her vision is presented and its application for VR is exemplified. The blur in human vision is considered as a combined effect of a number of processes involved in vision. Therefore the visual system is modeled as a complex system with probabilistic properties. As the work provides a theoretical base for modeling the blur it is a significant step to understand the phenomenon in detail and to introduce the concept in VR. We note that the probabilistic approach to perception presented in this work is complementary to the existing Bayesian approaches to perception, which are essentially statistical, in contrast to the present work, which is without statistical considerations. We show that the variation of blur is depending on the geometric relation between focus point and the second point considered, and we quantify this dependency. The dependency has apparently not been described before. The model of blur reveals that objects located in the region between the observer and the place where he/she is looking at are perceived sharper than objects located beyond the focal point. The corroboration of the theoretical results with common vision experience verifies the validity of the theoretical base of the work. In particular we note that the axiomatic starting point of the model development is verified. The model is applied to process the rendering of a VR scene for two different gazing acts of an observer. Based on the model outcome the degree of perceptual blur an observer would experience if the scene were physically present is simulated. The resulting perceptually blurred rendering of a virtual reality provides increased visual realism. This is because the visual stimulus provided by the perceptually blurred rendering matches more closely to the experience a human has when he/she would be experiencing the corresponding real environment, compared to a rendering where every pixel is equally sharp. A uniformly sharp image of a scene is an unrealistic representation of its physical counterpart in the sense that it requires excessive memory to hold all the details of the scene in case of a human observer. Application of perceptual blur in VR may contribute to alleviating the phenomenon of distance underestimation in VR and may support the provision of a more realistic spatial experience in VR, in particular with respect to large objects or spaces. Interestingly, in the present implementation increased realism is achieved not by increasing the amount of visual information of an image stimulus, but in contrary, by selectively reducing the information, as it apparently occurs in human vision system during an intentional act of looking.

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